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Efficient *F-K* Domain Wavefield Interpolation

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Abstract

Often, economic reasons dictate that seismic data be recorded with much larger spatial sampling intervals than basic sampling interval. This can cause harmful aliasing effects in prestack and poststack multichannel processing algorithms such as *f-k* filtering, Radon transform filtering, DMO, and migration. A common solution to sampling deficiencies is to use trace interpolation during processing. Among methods used in trace interpolation are sinc interpolation, dip search and slant stack, *f-k* domain deterministic dealiasing and the popular prediction filter based algorithms. Only the last class of algorithms can interpolate aliased temporal frequencies for steeply dipping events, but are expensive. Because of wrapping of aliased data inherent in the *f-k* domain, this domain was previously thought unsuitable for trace interpolation in the presense of steep dips. But, because of its efficiency, the *f-k* domain is an attractive domain to work with. This paper will present an efficient *f-k* domain data adaptive interpolation method that overcomes the problems in aliased data. The capabilities to interpolate steeply dipping events, to handle curvature by spatial windowing and to help in the removal of multiple interference will be illustrated. This new *f-k* wavefield interpolation is now used in production to minimize the aliasing effects in 3-D prestack multichannel processing.

Introduction

The recorded wavefield is a sampled version of the continuous wavefield and field geometry serves as a digital sampling process. Coarse digital sampling is known to cause aliasing. Ideally, shot, receiver, and temporal sampling intervals should be selected so that it is possible to construct a continuous wavefield from the recorded samples. The data are then said to be "properly sampled." The resulting sampling intervals are called "basic sampling intervals[1]."

Experience has shown that harmful aliasing effects in prestack and poststack multichannel processing result from data recorded with a much larger spatial sampling interval than the basic sampling interval.

Even when the signal (primaries) is properly sampled, it is quite likely that organized noise such as ground roll, mud roll, and multiples is undersampled, especially in areas where the noise or multiple energy has a much lower moveout velocity compared to the signal. With the improved dynamic range of current field equipment, it is common practice to use smaller arrays in the field and deal with the noise during processing. Although NMO correction is sometimes used to reduce the aliasing problem, for some recording geometries such as off-end marine shooting, an inherent aliasing problem remains in the common offset and common midpoint domains, even when shot and receiver domains are properly sampled. This aliasing problem remains because when the shot interval is equal to the receiver interval, trace distance in the CMP domain is twice that of the receiver interval. Shot intervals that are larger than receiver intervals further compound the aliasing problem. A popular solution to these sampling deficiencies is the use of interpolation during data processing.

Interpolation Techniques

One of the earliest interpolation techniques used in digital seismic data processing is the (sin x)/x, or sinc interpolation. This interpolator is well known in static correction applications and is used to shift traces along the time axis. Sinc interpolation assumes that the data do not contain higher frequencies than the Nyquist frequency imposed by the sampling interval. Although the sinc interpolator can be used to decrease the temporal sampling interval of the data, it cannot increase the temporal frequency content of the data. Therefore, sinc interpolation is known as a "band-limited" interpolation.

If the data are spatially band limited, the sinc interpolator can also be used along the space direction. Such an interpolation is accomplished at constant time samples, or in the *f-k* domain by zero padding along the wavenumber axis and inverse transforming. Sinc interpolation has recently been used for trace interpolation by Jacubowicz[2]. Regardless of the domain of application, sinc interpolation cannot increase the spatial frequency content of the data. If data are aliased, thereby wrapping the 2-D spectrum, the sinc interpolation leaves the spectrum unchanged, which represents a limitation of the method.

In the early 1980s, many local dip search and local slant stack types of algorithms that attempted to overcome aliasing problems emerged. However, these processes were nonlinear because they involved thresholding or semblance weighting schemes. Results depended on the sophistication of the particular algorithms. Interpolated traces did not quite match the original traces.

Prediction error filter-based algorithms, whether in the *f-x* domain[3] or in the *t-x* domain[4], can interpolate aliased data. The *f-x* domain algorithm relies on the low-frequency content of the data to interpolate a given frequency, assuming that the record is comprised of a limited number of linear events. It is an unaliased interpolator.

The *f-x* domain or its 3-D extension can be used to interpolate prestack seismic data. However, because of the size of the linear equations to be solved this technique becomes costly.

Because of the wrapping of aliased data inherent in the *f-k* domain, the *f-k* domain was previously thought unsuitable for steeply dipping events. But, due to its efficiency, the *f-k* domain is an attractive domain within which to work. Vermeer[1] proposed an *f-k* domain interpolation which can only handle dips not exceeding two time samples per trace.

However, we recently developed an unaliased *f-k* domain trace interpolation[5], UFKI, that overcomes the problems inherent in steeply dipping data. This 2:1 traces interpolator is derived as an all pass operator in the *f-k* domain of the input. This operator is a full band interpolator in the *f-k* domain of the final output (interpolated traces interlaced with interpolated traces)[5].

Unaliased *f-k* Domain Trace Interpolation

In the UFKI method the *f-k* transform of the unknown traces, U(f,k), is obtained by multiplying the *f-k* transform of the known data, K(f,k), with an operator, H(f,k),

$$U(f,k)=H(f,k)\,K(f,k) \qquad (1)$$

The required operator can be shown to be the ratio of "stretched" *f-k* transforms of certain components of the known data[5]. More explicitly, this operator is the ratio of stretched *f-k* transforms of even and odd numbered traces of the known gather:

$$H(f,k) = \frac{K_{even}\left(\frac{f}{2},\frac{k}{2}\right)}{K_{odd}\left(\frac{f}{2},\frac{k}{2}\right)} \qquad (2)$$

Stretching can be done by a simple interpolation like the 3-point quadratic interpolation filter (-1/8, 6/8, 3/8) along the f and k axis.

The UFKI operator as developed above is a one-sided (forward) interpolation operator. In the present paper we present a two-sided operator (forward-backward) which is indeed obtained from the one-sided operator. Since one needs to reverse the procedure to obtain odd numbered traces from the even numbered traces and since this produces the same data (except a lateral shift of one trace distance of the input record) the two-sided required operator is

$$H_{FB}(f,k) = \left(H(f,k) + e^{j2\pi k/N}\, H^*(f,k) \right)/2 \qquad (3)$$

$$= e^{j\pi k/N}\, \mathrm{Re}\,\{ e^{-j\pi k/N}\, H(f,k) \} \qquad (4)$$

For example, if the input is a single dipping event of slope ΔT seconds per trace then

$$H(f,k) = e^{-j2\pi f \Delta T/2} \qquad (5)$$

and

$$H_{FB}(f,k) = e^{j\pi k/N} \cos 2\pi \left(\frac{f\Delta T}{2} + \frac{k}{2N} \right) \qquad (6)$$

where N is the total number of wave numbers present in the Fourier transform. The complex exponential in the last expression corresponds to a half trace shift in the space direction and is the well known sinc interpolator[5]. Therefore we observe that the forward-backward interpolation operator inherently combines the data independent sinc operator with a data dependent component, the cosine term. The cosine term, in a sense, acts like a cosine taper along the k-direction; it is one at the ridge of the dipping event, i.e. when

$$k=k_0 = -Nf\,\Delta T \qquad (7)$$

and it goes to zero as k goes away from the peak by $K_{nyquist}$ amount (it is cyclic). We found that the filtering action (modulation to be more correct) of the forward-backward operator reduces interpolation artifacts. Note that this filter does not harm the data because its response is broader than the response of the data but suppresses energy at locations where there is no data. Note also that the t-x response of the forward-backward interpolation operator has the desirable symmetry around the origin (center of operation):

$$H_{FB}(-t,-x) = H_{FB}(t,x) \qquad (8)$$

This way traces before and after a particular interpolated trace give equal contributions along existing dip directions.

As pointed out before[5], when the data are not aliased one can do sinc interpolation alone. Sinc interpolation is much faster. Knowing the maximum dip in the data we can also determine the lowest frequency that will alias and replace lower frequencies with the sinc interpolator, resulting in a hybrid operator. This hybrid operator whether it is a forward or forward-backward operator is a data adaptive operator and will be referred as "the unaliased *f-k* interpolator" for the rest of the paper.

Examples

Figure 1 shows a two-event model with two opposite dips of -16 ms and 8 ms per trace. The sample interval is 4 ms. There are 30 traces in the gather and each event is a spike. The *f-k* spectrum of this gather (Figure 2) shows that both events are aliased. The sinc interpolation of this model (not shown) has the horizontal alignment of events. The *f-k* spectrum of the sinc interpolation has the same spectra for both the interpolated and the original gathers (band limited and wrapped). Figure 3 shows the result after unaliased *f-k* interpolation. The unaliased *f-k* interpolation is similar to the *f-x* interpolation method in that interpolated traces are obtained from original traces, making use of half frequencies in the original record. Therefore, it is important that the data include the low end of the spectrum. The unaliased *f-k* interpolator extends the spatial frequencies and unwraps the *f-k* spectrum once per application as shown in Figure 4.

Unaliased *f-k* interpolation is based upon two underlying assumptions – events are linear in the t-x domain, and there are a limited number of events in the 2-D Fourier transform window. When events have severe curvature, spatial gates help linearize the events.

The Unaliased *f-k* interpolator can be applied in either one of the following domains: common shot, common receiver, common offsets, common midpoint. In common midpoint application it produces shots halfway between the original shots. To help reduce the moveout range with very long offsets one needs to apply an NMO with a velocity function between primary and multiple trends. Figure 5 shows such a 44 fold gather from a CMP line from a 3-D marine survey.

Shot sampling interval in this recording was four times that of the receiver sampling interval, resulting in an 8 CMP periodicity in the offset distribution of the CMP gathers, causing noise from multiples to leak into the stack shown in Figure 7.

Figure 6 shows a cascaded application of unaliased *f-k* domain interpolation to these field data resulting in a 4:1 interpolation necessary to remove some of the multiple interference seen on the raw stack in Figure 7. The clean stack of the interpolated traces is shown in Figure 8.

Conclusion

incentive to acquire 3-D seismic data as economically as possible sometimes leads to spatial aliasing of the data which becomes difficult to process with multichannel algorithms. Trace interpolation has proven to be a useful method for solving this problem. In this paper an unaliased *f-k* domain trace interpolation which is symmetric around the point of application in the t-x domain was presented. This efficient algorithm was shown to help in suppressing harmful interference of multiples on the stack of a 3-D marine survey, when applied on the CMP gathers.

References

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3. Spitz, S., 1991, Seismic trace interpolation in the *f-x* domain: *Geophysics*, **56**, 785-794.
4. Claerbout, J. F. and Nichols, D. 1991, Interpolation beyond aliasing by (t-x) domain PEFs: EAEG 53rd Mtg., Expanded Abstract, paper A001.
5. Gulunay, N. and Chambers, R. E., 1996, Unaliased *f-k* domain trace interpolation (UFKI): 66th Ann. Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts.

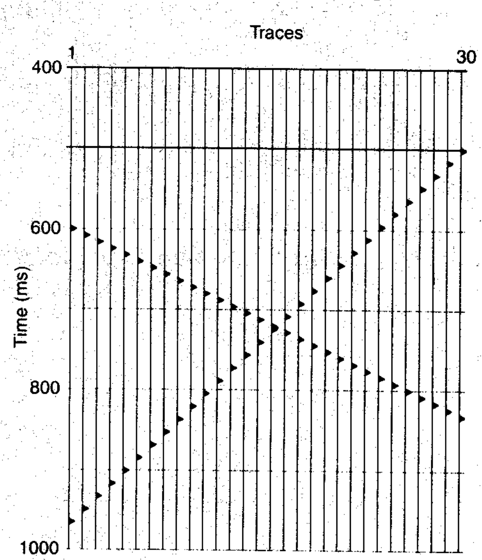

Fig. 1 - Input gather

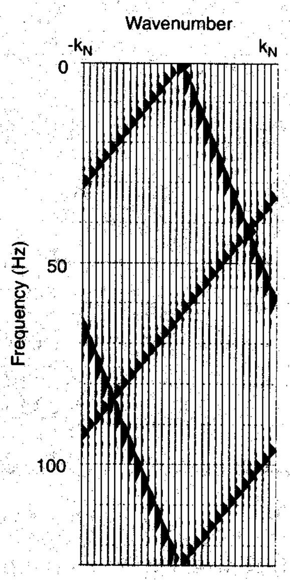

Fig. 2 - The *f-k* amplitude spectrum of the input gather.

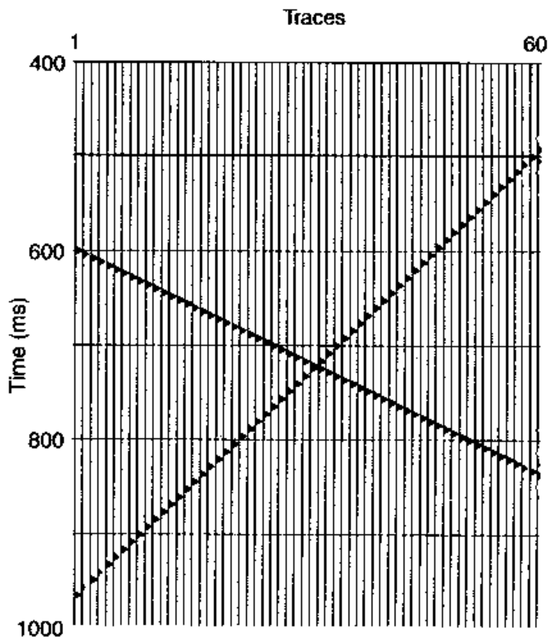

Fig. 3 - Unaliased *f-k* domain trace interpolation.

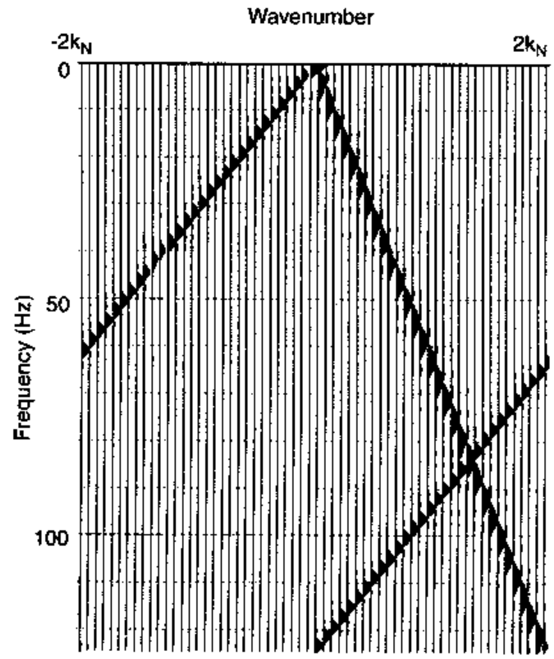

Fig. 4 - The *f-k* amplitude spectrum of the unaliased *f-k* trace interpolated gather.

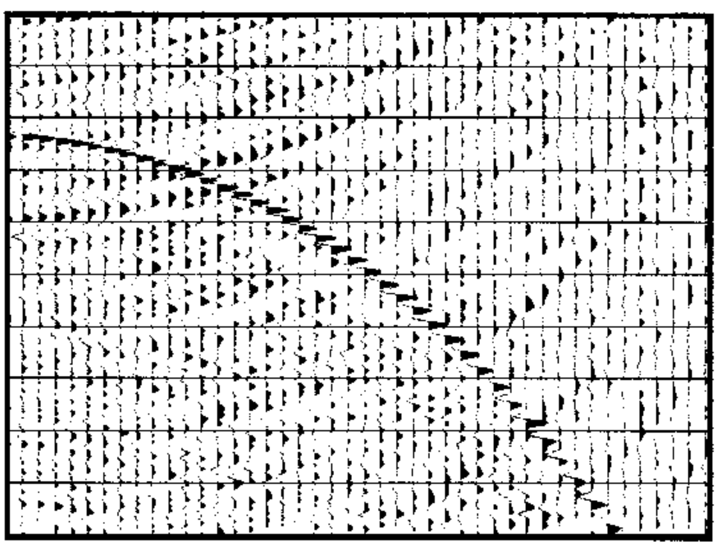

Fig. 5 - A 44 fold CMP gather after NMO with a velocity function between primary and multiple trends.

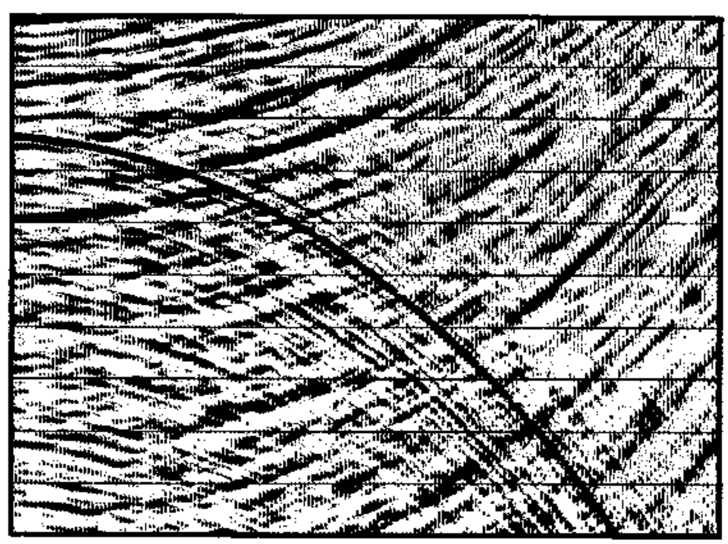

Fig. 6 - The CMP in Figure 5 after two cascaded application of UFKI resulting in a 4:1 interpolation.

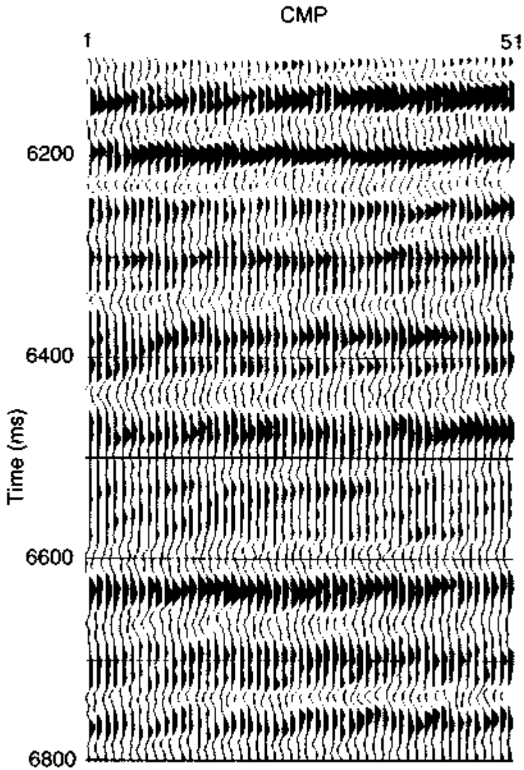

Fig. 7 - Stack of the 51 CMP gathers from a marine survey. The first CMP is shown in Figure 5.

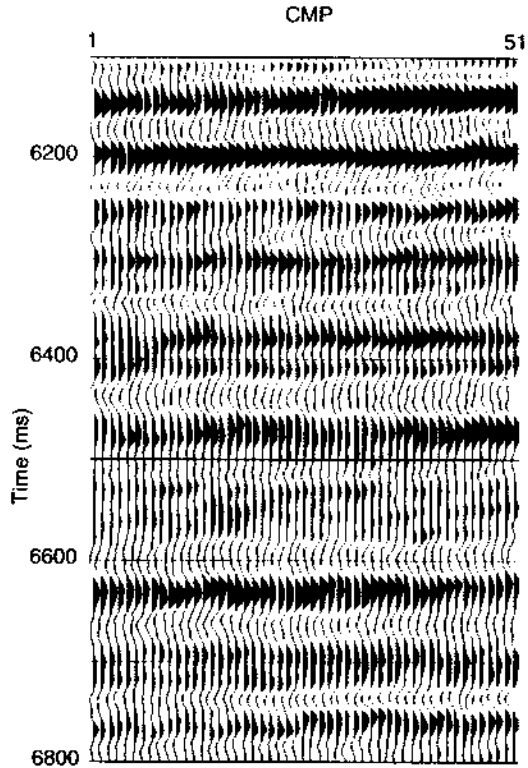

Fig. 8 - Stack after 4:1 unaliased *f-k* interpolation. The first interpolated CMP is shown in Figure 6.