Unaliased f-k domain trace interpolation (UFKI)

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Summary

Trace interpolation for aliased events may be achieved efficiently by working in the f-k domain where the process unwraps thef-k transform of aliased events. After unwrapping, multichannel processes that are sensitive to spatial aliasing, such as *f-k* filtering, Radon transformation filtering, DMO, and migration, produce more desirable results.

Introduction

The aliasing that results from inadequate digital sampling is well known. Interpolation of digitized and aliased one-dimensional data without aliasing artifacts is impossible; however, interpolation of two-dimensional aliased data may be achieved for data containing a finite number of events which approximate straight lines on 2-D displays. In the following sections, we review interpolation related work for both one and two dimensions.

Aliased 1-D signals

Although sampling digital signals to a new set of unit delay samples is a simple index shifting, resampling with fractional delay is not trivial. Fractional delay, i.e., interpolation, of one-dimensional digital signals is achieved by making assumptions as to the spectral content of the data. A common assumption is that underlying signal does not have a spectrum wider than that of the original samples,

$$D(k') = D_{known}(k') \quad \text{for } |k'| < k_N$$
$$= 0 \quad \text{otherwise},$$

where $D_{known}(k)$ represents the Fourier transform of the original samples (known data) and k_N is the Nyquist frequency. Note that k' is the wavenumber of the interpolated gather and its range is larger than that of k by a factor, M, corresponding to the value of fractional delay

$$k'_N = Mk_N \quad .$$

When fractional delay is 0.5, M is equal to 2.

This assumption of band limitedness (to the baseband) of the interpolated data insures that interpolated samples behave well when placed between the original samples. When only the interpolated samples are considered, the best one can hope to perform is an all-pass operation in the original transform domain:

$$|D_{unknown}(k)| = |D_{known}(k)| \quad .$$

While many 1-D interpolation methods are available, the best such methods can achieve is to approach the ideal filter. The ideal filter is all-pass, but band-limited when interpolated samples are interleaved with original samples (Laakso et al., 1996). This ideal filter is known as sinc or sine cardinal function. When the fractional delay is equal to half of the sample interval, the sinc function becomes

$$h(n) = \frac{\sin(\pi(n-0.5))}{\pi(n-0.5)} \quad \text{integer } n \quad .$$

Its system response is

$$H(k) = e^{j\pi\frac{k}{N}} \quad \text{integer } k \quad .$$

When the underlying signal has a finite bandwidth and when the sampling rate is higher than the maximum frequency contained in the signal, the sampled version of the signal is adequate for reconstructing the signal values at any fractional delay. On the other hand, when either the signal is not band limited or sampling is course, the 1-D signal is aliased and there is no unique way of reconstructing the interpolated samples. Shannon's sampling theorem (Marks, 1991) states lack of uniqueness in a more rigorous fashion. For this reason, analog signals are passed through an analog high-cut filter before being digitized.

Aliased 2-D signals

Two-dimensional digital signals suffer from similar aliasing effects with one difference: the aliased 2-D spectrum can be unwrapped accurately if the underlying data are made of a finite number of linear dipping events.

It has long been recognized that multidimensional seismic data processing algorithms, such as migration and DMO, are prone to potentially damaging artifacts due to aliasing. These processes do no damage if data going into them are properly sampled. When required samples are not recorded in the field, we need to reconstruct them from the field data. Many of our industry's poststack interpolation algorithms emerged from this need for proper sampling.

The most straightforward trace interpolation method is a 1-D interpolation in the space direction at each time sample by Fourier transforming in the *k* direction, padding a large enough number of zero samples, and inverse transforming. Of course, . the same interpolation is possible for each frequency slice of data if one prefers to work in *f-k* domain. The process is the deterministic, data independent, sinc function, trace interpolator. We find that sinc interpolation, which is performed by zero padding along the *k* axis, can be expressed in a different way when the *f-k* transform of the unknown traces need to be related to the *f-k* transform of the known traces:

$$D_{unknown}(f,k) = e^{j\pi\frac{k}{N}} D_{known}(f,k) \quad .$$

That is, the sinc interpolator may be written as

$$H(f,k) = e^{j\pi\frac{k}{N}} \ ,$$

and is independent of the temporal frequency, f Note also that the interpolator is an all pass operator in the input domain. When unknown traces are interleaved with the known traces, the spectrum of the new gather has the same content as the known traces (half-band interpolation in k'). This spectrum is confined to $-k_N$, $+k_N$ zone and is wrapped.

Unaliased *f-k* interpolation

Sinc interpolation does not solve the interpolation problem adequately when data are spatially aliased. Many interpolation algorithms are available within the industry but most suffer from aliasing. In some approaches, aliased energy is removed by filtering when possible. Spitz (1989, 1991), however, exactly honored and interpolated aliased data using a temporally wideband assumption and scarcity in the number of dips the data contain. His method is known as *f-x* interpolation. Spitz's work inspired further research at Stanford University (Ji, 1991; Balog, 1991).

How field geometry (source and receiver positions) of seismic recording forms a digital surface sampling and how field arrays can help suppress spatial aliasing by limiting the spatial bandwidth of the signals are discussed in detail by Vermeer (1990). The use of field arrays affects both signal and noise. More importantly, Vermeer points out that trace interpolation in the common midpoint domain can de-alias the seismic wavefield by providing missing shots. An interpolation of this type may reduce, among other things, the checkerboard effect that some land geometries exhibit on time slices or the zipper effect caused by multiple leakage seen on multisource multicable marine data. Vermeer also offers a limited unwrapping scheme (from first half *k* plane to third half *k* plane) when input data are recorded off-end and all moveouts are of the same sign.

Poststack suppression of such artifacts is demonstrated to be successful in limited cases (Gulunay et al., 1994) by K_x-K_y or simply K-notch filtering (Hampson, 1995). Aliasing problems are best addressed prestack either within the multichannel algorithms such as DMO (Beasley and Mobley, 1988) or by interpolation before the multichannel process.

Because the normal moveout (NMO) process can reduce aliasing to a large degree, Jacubowitz (1994) was able to interpolate prestack traces via the sinc interpolator in wavefield reconstruction. When the residual moveout left after NMO is not severe (i.e., not much conflicting dips or multiples in the data), the sinc interpolator works well.

Wombell and Williams (1995) use Vermeer's idea of shifting from first half *k* plane to third half *k* plane (for 3-D data) and do prestack interpolation in the f-k_s-k_r domain (frequency, shot wavenumber and receiver wavenumber domain).

Recently Mannin and Spitz (1995) used the unaliased f-x interpolator in the CMP domain to achieve wavefield reconstruction. Unfortunately, solving the two sets of linear equations in the Spitz method is expensive, especially if one uses the edge-effect-free form of the prediction equations which lead to non-Toeplitz matrix structure.

Claerbout (1991) provides an unaliased interpolator but in the t-x domain through 2-D prediction error filters. We did not test his interpolator but expect it to be costly.

Among all interpolators discussed above, the accurate ones in the presence of aliased energy are *f-x* or *t-x* prediction error filter methods. Because of cost considerations we explored the *f-k* method. We find that trace interpolation can be done adequately in *t-x* domain. Our method is limited by the same factor as Spitz's method: data must not be missing low frequencies (see Balog, 1991). In the following, we explain our method and tie it to Spitz's method.

Unaliased *f-k* interpolator (UFKI)

Basic concepts leading to the unaliased *f-k* interpolator for fractional delay of 0.5 along the space direction are given below.

1) *Odd and even numbered traces of a gather each have an f-k transform which is wrapped once compared to the transform of the original gather.* *F-k* transform of the odd and even numbered traces can be calculated from the *f-k* transform of the original gather using expressions given by Burrus (1985).

2) *Interleaving odd and even numbered traces is a process which unwraps the spectrum of odd (or even) numbered traces once.* Therefore, if we are able to calculate even numbered traces properly, interleaving them with the original (odd numbered) traces will produce an unwrapped (full *k* band) spectrum. In other words, a proper interpolation of 2-D data extends the spectrum from $-k_N$, $+k_N$ range to $-2k_N$, $+2k_N$ range. A correct interpolator is a full-band interpolator. rather than half-band (sinc) interpolator.

3) *The magnitudes of the f-k transforms of the odd and even numbered traces of a gather are almost identical.* Therefore, we can assume that the unknown *f-k* transform differs from the known *f-k* transform only by phase.

4) *Phase information needed to reconstruct the unknown traces from known traces can be obtained from the f-k transform of certain components of the known gather at half temporal frequency.*

Our method can be related to Spitz's method as follows. When one writes the Spitz equations for unknown data, one finds that it splits the prediction error filter coefficients into odd and even numbered components. We consider only forward (or backward) prediction equations and keep only the equations which contain one kind of prediction error filter coefficients on one side of the equations (i.e., odd numbered components on the left and even numbered components on the right or vice versa). We assume that resulting linear equations can be converted to cyclic convolutions. With these approximations, one can replace the Spitz equations with complex number products in the *f-k* domain. In other words, we consider only part of the Spitz equations

$$(e_1, e_3, \ldots) * (x_2, x_4, \ldots) = -(e_2, e_4, \ldots) * (x_1, x_3, \ldots)$$

(where e_1=1) as Balog (1991) did. We then express these equations in the *f-k* domain:

$$D_{unknown}(f,k) = \frac{B(f,k)}{A(f,k)} D_{known}(f,k)$$

Coefficients *A(f,k)* and *B(f,k)* belong to the left and right hand side, respectively, of the previous equation. They are related to the Fourier transform of the original gather. This is so because prediction error filter coefficients $(e_1, e_2, e_3, e_4, \ldots)$ are obtained from known data at half temporal frequency. The *f-k* transform of the prediction error filter E relates to the *f-k* transform of known data because

$$E(\frac{f}{2},k) = \frac{1}{D_{known}(\frac{f}{2},k)}$$

Note that the operator H = $B \,/\, A$ can be considered as a deconvolution operator applied in the *f-k* domain. Some white noise addition is necessary to protect against division by very small numbers. Note also that operator H depends generally on both *f* and *k*. In the special case of a single dip event the operator reduces to

$$H(f,k) = e^{\,j\, 2\pi f(\Delta t/2)}$$

where At is the slope (seconds per trace) of the dipping event on the input gather. Note that this full-band, unaliased, *f-k* interpolator is independent of wavenumber *k*. This is in contrast to the sinc interpolator, which is independent of temporal frequency *f*.

Model data example

Figure 1 shows a 30 trace 4 ms sampled model that includes two aliased events (both are full-bandwidth), one with a dip equal to two sample intervals per trace, the other with a dip equal to -4 sample intervals per trace. Figure 2 is the result of a zero padding type (sinc interpolator) *f-k* interpolator. Note that odd numbered traces are identical to the original data. Both events show aliasing artifacts on even numbered (interpolated) traces. Figure 3 shows the result of the unaliased *f-k* interpolator. Again, odd numbered traces are identical to those of the input data and interpolated traces are the even numbered traces. We see that they now tie to the original traces well.

Field data example

UFKI as cast above is only valid for linear events as are other spatial interpolators. We can apply UFKI to complicated data if we consider small data windows. To eliminate the artifacts (operator wraparound), we may need to consider operator tapering in the *t-x* domain. If interpolation is done in the time-offset (CMP, common shot, or common receiver) domain, we must apply NMO to reduce the curvature existing on the data. With these precautions, satisfactory results can be obtained for aliased data. Figures 4 and 5 show before and after band-limited (sinc type) *f-k* interpolation processes on an NMO-corrected CMP gather. Note that the NMO function was chosen to be between the primary and multiple trends so that both could be interpolated equally well. We observe that sinc interpolator aliases at far offsets. Figure 6 is the result after unaliased f-k interpolation. We observe that aliased events are correctly interpolated by UFKI.

Conclusions

We have demonstrated that it is possible to interpolate aliased seismic gathers using only the *f-k* domain. In this domain, solution of linear equations reduce to complex number divisions.

Acknowledgments

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References

Balog, O., 1991, 2-D interpolation with Spitz's method: SEP-72,19-3 1.

Beasley, C. and Mobley, E., 1988, Amplitude and antialiasing treatment in (x-t) domain DMO: 58th Ann. Intemat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1113.

Burrus, S., 1985, DFT/FFT and convolution algorithms: John Wiley & Sons, Inc.

Claerbout, J., 1991, Earth sounding analysis: processing versus inversion: Blackwell Scientific Publications, Inc.

Gulunay, N., Martin, F., Martinez, R., 1994, 3-D data acquisition artifacts removal-spot editing in the spatial-temporal frequency domain: EAEG 56th Mtg., Expanded Abstract, paper H049.

Hampson, G., 1995, Relationship between wavefield sampling and coherent noise attenuation in the CMP method: 65th Ann. Intemat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 939.

Jacubowitz, H., 1994, Wavefield reconstruction: 64th Ann. Intemat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1557-1560.

Ji, J., 1991, Trace interpolation using spectral estimation: SEP-72,57969.

Laakso, L., Valimaki, V., Karjalainen, M., and Laine, U., 1996, Splitting the unit delay: IEEE Signal Processing, 44, no. 1, 30-60.

Mannin, M. and Spitz, S., 1995, Wavefield de-aliasing for acquisition configurations leading to coarse sampling: 65th Ann. Intemat. Mtg., Soc. Expl. Geophys., Expanded Abstract, 930.

Marks, R. J., 1991, Introduction to Shannon sampling and interpolation theory: Springer-Verlag New York, Inc.

Spitz, S., 1989, Trace interpolation beyond aliasing and without picking: 59th Ann. Intemat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 1121.

Spitz, S., 1991, Seismic trace interpolation: Geophysics, 50, *785-794*.

Vermeer, G. J. O., 1990, Seismic wavefield sampling: SEG Publications.

Wombell, R., and Williams, R.G., 1995, Aliasing and sampling problems for multisource and multicable acquisition, Exploration Geophysics, 26,472.476.

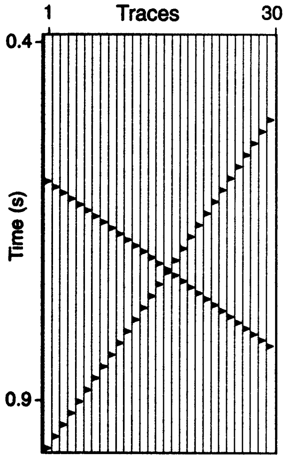

Fig. 1: A simple model with two aliased events.

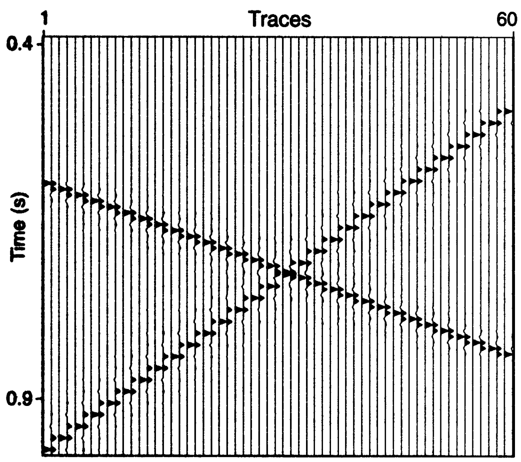

Fig. 2: The model after band-limited (i.e., sinc type) *f-k* interpolation.

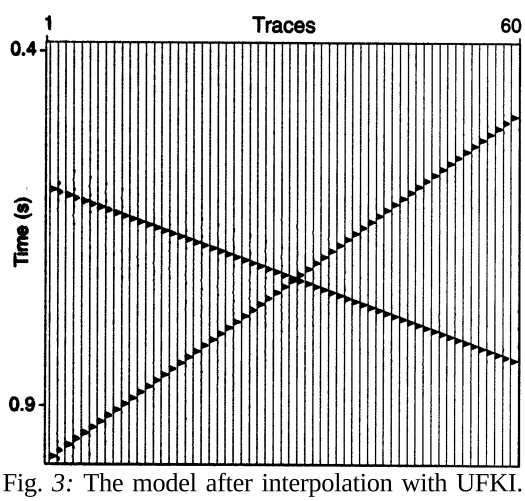

Fig. *3:* The model after interpolation with UFKI.

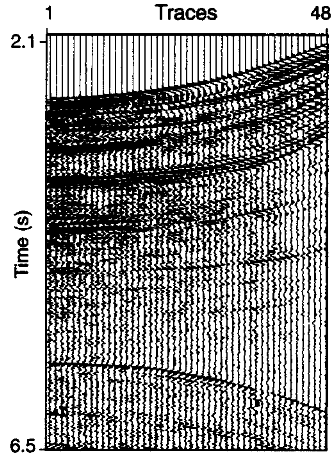

Fig. 4: A CMP gather (NMO-corrected with a velocity function between primary and multiple velocity trends).

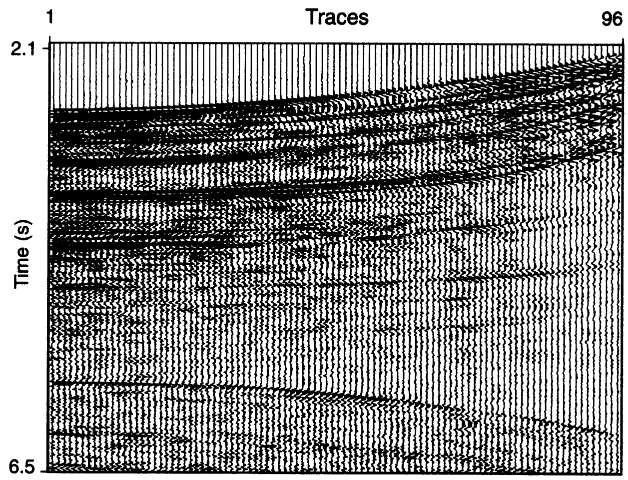

Fig 5: The CMP gather after sinc type *f-k* interpolation

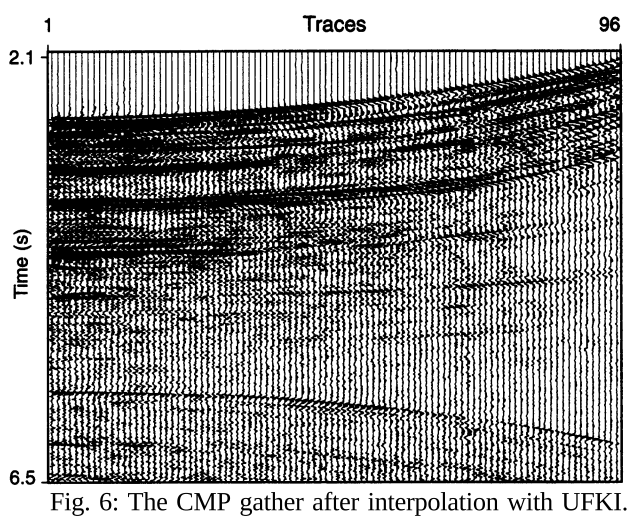

Fig. 6: The CMP gather after interpolation with UFKI.