Prediction Filtering for 3-D Poststack Data

SP4.3

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SUMMARY

In designing operators for random noise suppression the following factors are important: uniqueness of operators, insensitivity of the operators to the size of the gate used in their derivation, shortness of operators in all directions, abrupt event termination handling and amplitude variations handling.

In the frequency domain spatial prediction filtering, operators are generally designed as forward prediction filters through the use of Yule-Walker equations. They are then halved, conjugate flipped and placed in front of the forward filter with one zero in between. The resulting operator is zero phase. We will call such operators "pseudo-forward-backward operators". As demonstrated in this work, true forward-backward prediction filters (designed using noncausal normal equations) seem to satisfy all of the above criteria much more than their pseudo forward-backward counterparts.

INTRODUCTION

Prediction filtering in two dimensions has recently been used for post stack 3-D data (Chase, 1992). Filters of this method are different but similar to pseudo forward backward filters. We use a true forward-backward (ie. noncausal) design technique which has some interesting properties. Before introducing our method we review the previous work on one dimensional forward prediction filters used in the industry. We then introduce one dimensional forward-backward filters.

a)- One dimensional forward prediction filters for 2-D post stack data

One dimensional post stack random noise elimination filters were first introduced by Canales (1984). This approach follows Claerbout (1976), and uses auto-correlation lags $(r_0, r_1, \ldots, r_L)$ for the prediction error filter $(1, e_1, \ldots, e_L)$. Resulting prediction error operators are minimum phase in the space direction (forward). The normal equations resulting from this approach are also known as (pre- and post-) windowed Yule-Walker equations (Kay and Marple, 1981) and can only predict "impulsive" (minimum phase) inputs or impulsive components of the input (impulsive series are also known as autoregressive or AR processes). The normal equations for the prediction error filter involve auto-correlation lags $(r_0, \ldots r_L)$ on the left-hand side of the normal equations and a spike at zero lag on the right-hand side. (The corresponding normal equations for the prediction filter $(p_1, \ldots, p_L)$ involve auto-correlation lags $(r_0, \ldots r_{L-1})$ on the left-hand side and lags $(r_1, \ldots r_L)$ on the right-hand side of the normal equations.) Since each auto-correlation lag involves one less term than the previous lag, small data gates, say 10 traces, give unacceptable prediction error filters with this technique. That is, this approach can't even predict one event exactly if the data gate is short(Harris and White, 1991).

Gulunay (1986a) similarly used windowed auto-correlations but a different approach to the solution (complex Wiener filter design). He observed that the shortness of the desired output by one sample caused significant prediction errors for small space gates and suggested a hybrid method named "fxdecon" (now used as a generic name) which keeps an extra trace to use in forming the cross-correlation on the right-hand side of the normal equations. The resultant filters are not necessarily minimum-phase. This approach reduces the gate effect to zero for the special case of one event.

b)- One dimensional noncausal prediction filters for 2-D stack data

Because dips are handled properly during filter design, there is no danger of moving events laterally with forward prediction filters. However, residues from unpredictable components, such as sudden truncations in data, move energy forward with the application of forward filters. Therefore it is important to do the processing either both in forward and reverse directions (Gulunay, 1986b) or with zero phase filters in the space direction. This requirement is known to be equivalent to conjugate symmetry in the filter: $P_{-k} = P_k^*$ for $k = 1, \ldots, L$. In going from one-sided (causal) prediction filters

$$(p_1, p_2, \ldots, p_L)$$

to two-sided (noncausal) filters the usual procedure is to form the operator

$$0.5(p_L^*, \ldots, p_1^*, 0, p_1, \ldots, p_L)$$

where * indicates complex conjugation. We will call such operators *pseudo FBPF* (forward-backward prediction filter).

For an input sequence containing L linear events, the use of a 2L+1 length (noncausal) operator and the use of noncausal normal equations result with the *true FBPF,* say,

$$(P_{-L}, \ldots, P_{-1}, 0, P_1, \ldots, P_L)$$

The normal equations are of size 2L+1 and involve auto-correlations lags $r_0, \ldots, r_{2L}$. The matrix representing the coefficients is Toeplitz Hermitian and positive definite because of the windowed nature of the auto-correlation lags. The solution is conjugate-symmetric ($P_{-k} = P_k^*$) as is the pseudo FBPF. However, true FBPF sample values are different from those of pseudo FBPF and have smaller prediction errors on the pure signal. This will be shown in the following example. Consider a 5-point one event input $(1, z_0, z_0^2, z_0^3, z_0^4)$. The 3-point pseudo FBPF is $(0.4z_0^*, 0, 0.4z_0)$ and its error series is $(0.6, 0.2z_0, 0.2z_0^2, 0.2z_0^3, 0.6z_0^4)$. The 3-point true FBPF is $(0.5z_0^*, 0, 0.5z_0)$ and its error series is $(0.5, 0, 0, 0, 0.5z_0^4)$. (Non-zero coefficients here represent unpredictable parts and can be ignored.) We observe that true FBPF is not sensitive to the gate size it is derived from. Note that the prediction filter designed by Gulunay's method mentioned above produces forward prediction filter z,. The pseudo FBPF that will be derived from it is identical to the true FBPF

$$(0.5z_0^*,\ 0,\ 0.5z_0)$$

in this case (but not so for other cases). Note that for one event input of any length larger than 3, the filter is always the same. This illustrates the insensitivity of the filter to the gate length. Note also that for noise free input, the filter coefficients do not vary significantly if the length of the filter is increased. This is due to the positive definiteness introduced to the normal equations by the windowed nature of the auto-correlations.

One dimensional true FBPF filters can be also shown to preserve the correct output amplitude level in inputs which show smooth variation of amplitudes by spatial direction. For example, a dipping event with exponential amplitude variation in the space direction can be represented as $(1, a, a^2, \ldots, a^{N-1})$ where $a = e^{\alpha} z_0$. It can be shown that the 3-point pseudo FBPF obtained from Yule-Walker approach is

$$0.5e^{\alpha}\,\frac{S_{N-2}(\alpha)}{S_{N-1}(\alpha)}\left(z_0^*,\ 0,\ z_0\right)$$

where $S_n = 1 + e^{2\alpha} + e^{4\alpha} + \ldots + e^{2n\alpha}$. This filter produces large errors when exponential variation is large and when N is small. For example for $\alpha = 1$ and N = 30 we get 43 percent prediction error in all samples of the output (excluding the two samples at the edges where it is worse). On the other hand true FBPF designed as prescribed above is

$$0.5\frac{1}{e^{\alpha}+e^{-\alpha}}\left(z_0^*,\ 0,\ z_0\right)$$

and produces no prediction error except at one sample at each end of the window. This analysis suggests that a true FBPF will be more desirable for data with spatially varying amplitudes.

TWO DIMENSIONAL FORWARD BACKWARD PREDICTION FILTERS FOR 3-D POST STACK DATA

Multi channel least squares filters have been used in the geophysical industry for sometime. However, the use of two dimensional prediction filters for post stack random noise reduction is quite recent (Chase 1992). Multidimensional digital signal processing has found many applications in electrical engineering field, A number of papers are available on two dimensional prediction filters (IEEE, 1986). Such filters can be designed as causal, semi-causal Or noncausal filters. We chose the noncausal (2-D FBPF) approach because of the reasons explained above for 1 -D filters. Then, every output point is predicted by using a mesh of points around it *excluding* itself. We choose filter dimensions to be odd in each direction and use one set of normal equations for all the filter coefficients. The resultant filter coefficients have conjugate symmetry and the filter is zero phase in the spatial frequency space. For example, 3x3 prediction filter derived from a dipping plane gives

$$\begin{vmatrix} -0.25z_0^*w_0^* & 0.5z_0^* & -0.25z_0^*w_0 \\ 0.5w_0^* & 0 & 0.5w_0 \\ -0.25z_0w_0^* & 0.5z_0 & -0.25z_0w_0 \end{vmatrix}$$

where w_0 is defined similar to z_0 but for the second space dimension. The matrix involved in the solution of the normal equations is Hermitian, positive-definite. The equations can be converted into real form and solved by a real matrix solver like Gauss-Jordan method.

WHAT IS SO GOOD ABOUT TWO DIMENSIONAL FILTERS?

Two dimensional filters (let us call them l-pass prediction filters) are generally more costly than two passes (inline and crossline) of one dimensional filters. One may naturally ask if there are other benefits obtained by two-dimensional filters. Chase(1992) correctly pointed out that for certain structures such as cylinders 2-D filters do a better job than I-D filters. We would like to extend this conclusion by saying that as long as geology does vary along one of the axis (x or y), and when noncausal forward-backward prediction filter design is used, even the shortest filter (3x3) is sufficient to predict such geology with zero prediction error (in the absence of noise).

Some of such surfaces are given in Figure 1. For such surfaces data correlates in one dimension perfectly and the filter pulls most of its information about data from that dimension. When the input is a made of fan of dipping planes passing through one common axis we see that the prediction will fail if the length of the I-D filter is less than the number of planes present and if prediction is done perpendicular to the common axis. On the other hand, a 3x3 noncausal filter predicts this structure reqardless of the number of dipping planes it may contain. It is also worth mentioning that a 3x3 noncausal filter passes vertical faults oriented along x or y direction with no distortion. If the faults are oriented in an oblique direction in the x-y plane a longer filter such as 5x5 will be needed and sufficient.

A simple 45 degree 3-D fault model is studied in Figure 2. We compare a 2-D filter application to two-pass 1-D filter application and observe that the 2-D filter preserves the fault where the two-pass 1-D filter application does not.

In Figure 3 we compare a one-pass (FXYP) run to a two-pass run on a faulted geology and observe that the one-pass run preserves data better than the two-pass run.

CONCLUSIONS

We have shown that true FBPF operators are less sensitive to the size of the gate they are derived from than pseudo FBPF operators. We have also shown that two-dimensional true FBPF operators are more compact than one dimensional filters needed for a two-pass run. This means two-dimensional filters are more likely to preserve spatial resolution. Any number of dipping planes with conflicting dips, and even a complex geology can be handled by a 3x3 Operator when geology does not vary in one direction. 2-D true FBPF operators preserve faults along in-line and along cross-line directions. Slightly longer operators. like 5x5. are needed when faults lie in an oblique direction. We have also shown that output levels of a true FBPF are more correct than that of a pseudo FBPF for an event exhibiting lateral amplitude variations.

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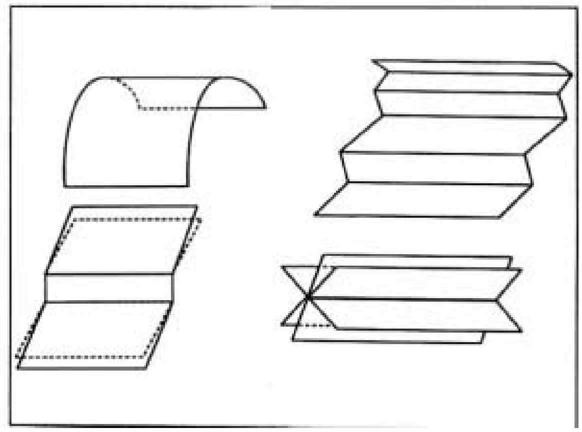
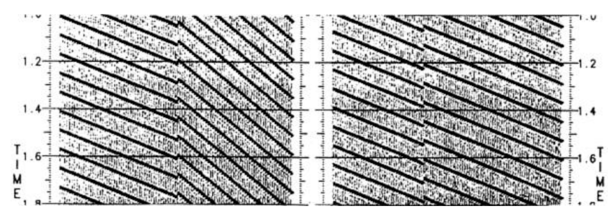


Figure 1 Some exactly predictable surfaces

3-D Fault model

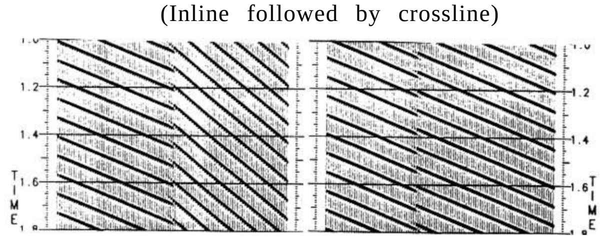
An in-line and a cross-line

INPUT



Two-pass (FX)prediction

(Inline followed by crossline)



One-pass (FXY) prediction

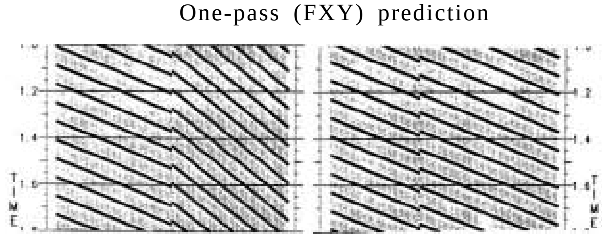


Figure 2 Comparison of a 5x5 true FBPF filter application (gate size 40x40) to a 5-point two-pass (gate size 401 pseudo FBPF application. Gate overlap is 33 percent.

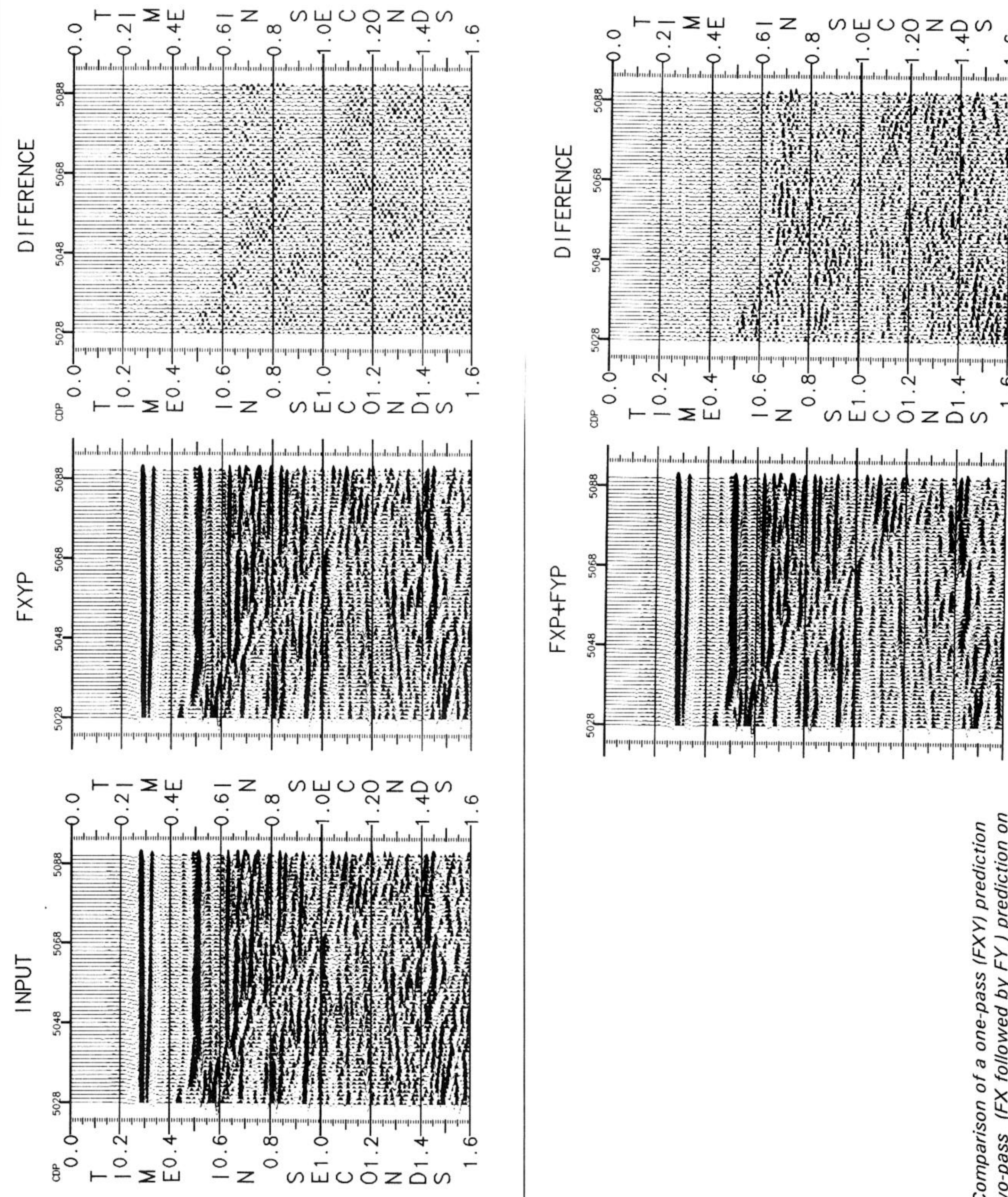


Figure 3 *Comparison of a one-pass (FXY) prediction run to a two-pass (FX followed by FY) prediction on a faulted geology.*