

TÜRKİYE 9. PETROL KONGRESİ
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9th PETROLEUM CONGRESS OF TÜRKİYE
UCTEA CHAMBER OF GEOPHYSICAL ENGINEERS
TURKISH ASSOCIATION OF PETROLEUM GEOLOGIST
UCTEA CHAMBER OF PETROLEUM ENGINEERS

TARİH (DATE): 17-21, 2, 1992

YER (PLACE): HILTON-ANKARA

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

NECATİ GÜLÜNAY, HALLIBURTON GEOPHYSICAL SERVICES, SUGAR LAND, TX.

ÖZET

Alışıl gelmiş hız analizi tekniklerinde zaman pencereleri ve bu pencerelerden türetilen "semblans" türünden ölçümler kullanılır. Yeni bir ölçüm olan kovaryans ölçümünün kullanılmasıyla hız analizinin zaman ve hız doğrultusundaki ayırım gücü kayıttaki gürültünün çok küçük olmadığı hallerde artırılabilir. Kayıtlardaki gürültünün çok az olduğu durumlarda tekniğin zaman yönündeki ayırma gücü hız yönündeki ayırma gücüyle çelişmektedir. Özellikle birbirleriyle etkileşecek kadar yakın iki olayın birarada olduğu durumlarda ne hız ne de zaman doğru olarak ölçülebilir.

Kovaryans matrisinin elemanları iki ize ait iki pencerenin kroskorelasyonundakisıfır gecikmenoktasının değerinden oluşur. Bu nedenle, kovaryans ölçümü enerjisi indirgenmiş kroskorelasyon toplamı ile yakından bağlıdır. Semblans ile enerjisi indirgenmiş kroskorelasyon toplamı birbirlerine yakından bağlı olduklarından sadece semblans kullanılarak ta kovaryans ölçümü tanımlanabilir. Bu şekilde üç ayrı türlü kovaryans ölçümü tanımlıyabiliriz.

Her üç ölçüm de benzer özelliklere sahip olup bu ölçümlerle yaptığımız deneyler aşağıdaki sonuçları vermektedir: a)- Sinyal/Gürültü oranı yüksekken Sinyal/Gürültü oranını temsil eden terim yeteri kadar hız ayırımı sağlar, b)- Sinyal/Gürültü oranı düşükken hız ayırımını logaritmik terim sağlar, c)- zaman ve hız ayırma yeteneği kullanılan zaman penceresinin büyüklüğüne, kayıttaki gürültü miktarına, bilgi işlemde kullanılan beyaz gürültüye, ve birbirleriyle etkileşen olayların var olup olmadığına bağlıdır, d)- bilgi işlemdeki beyaz gürültü kayıttaki gürültü gibi davranmaktadır, e)- logaritma teriminin üssü büyük seçilirse zayıf olayları yok etme tehlikesi oluşur.

ABSTRACT

Traditional velocity analysis techniques use time gates and attributes such as semblance to detect event velocity and zero offset time. The use of a recent attribute, covariance measure (CM) which is defined through the use of the eigenvalues of the covariance matrix, increases the resolution in time and velocity as long as the amount of noise in data is not small. If the noise is extremely small then temporal and velocity resolutions are in conflict and for interfering events neither velocity nor time is accurate.

Each element in the covariance matrix is the zero lag of the cross correlation between two gates belonging to a trace pair and therefore CM is related to energy normalized cross correlation sum. A covariance measure can be defined from energy normalized cross correlation sum. It is well known that semblance and energy normalized cross correlation sum are related. Therefore, a covariance measure can also be defined from semblance and is the most economical of the three methods.

Experiments with covariance measure further show the following:

a)-When signal to noise ratio is high the SNR term in CM is enough to give high velocity resolution, b)-When signal to noise ratio is low the log term in CM provides resolution, c)-Temporal and velocity resolution is affected by the time gate size, data noise, processing noise(white noise) and if there are interfering events or not, d)- processing noise acts like data noise, e)-Large exponent for log term is likely to suppress weak events.

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

INTRODUCTION

Velocity analysis techniques that use temporal gates map the time-offset plane into the time-velocity plane by assigning a coherency related attribute to the gate center time (t) and scan velocity (v).

Semblance, unnormalized cross correlation sum, statistically normalized cross correlation sum, energy normalized cross correlation sum (ENCCS) and some other attributes such as mean amplitude in the gate have been used (Neidell and Taner, 1971). Recently, new techniques which increase the resolution of the attributes have emerged: Sguazzero and Vesnaver(1986) and Sguazzero et al.(1987) used a complex coherency measure derived from statistically normalized complex cross correlation sums, and Key et al,(1987) as well as Biondi and Kostov(1988) used the eigenvalues of the covariance matrix. Following the work of Key et al, Key and Smithson(1990) chose "covariance measure" (CM) as the attribute. CM is defined by them as the product of two terms: signal to noise ratio (SNR) and a weight (W) which is the measure of the inequality of the eigenvalues of the covariance matrix. They reported CM to have better resolution than semblance and to be less sensitive to the statics problems in data. Also, "signal subspace" concepts such as dynamic modeling of the noise field were interesting. Because of such appealing properties of the covariance measure the present study was undertaken.

During this study several facts emerged:

a)- Signal to noise variance ratio defined by Key and Smithson(1990) is the same as signal to noise energy ratio calculated from semblance or from ENCCS.

b)- For high signal-to-noise ratio, the apparent high resolution in CM is mainly due to the use of signal to noise ratio, but not due to the use of eigenvalues per se. The resolution of the signal to noise ratio obtained from the semblance is same as the resolution of the covariance measure.

c)- For low signal-to-noise ratio, the resolution comes from the weight W. However, this weight which is obtained from the eigenvalues is not numerically stable for low signal-to-noise ratio and can give spurious peaks. Therefore, precautions taken to eliminate the numerical problems have a strong influence on the outcome. W factor is equivalent to raising semblance (or ENCCS) to a power such as N_x .

d)- For high signal-to-noise ratio, side lobes of the

wavelets give as high coherency values as the main lobe, but at different velocities from the main lobe. The velocity spectrum forms a ridge like shape. Due to this effect (Ridge Effect) both time and velocity resolution is lost. Especially for two or more interfering events, peaks don't occur at the right places in v,t plane. CM suffers from this effect as well.

e)- ENCCS is intimately related to the eigenvalue technique of CM, and a theoretical CM can be obtained from ENCCS without solving for the eigenvalues.

f)- Since ENCCS and semblance measure very much the same quantity, CM can be calculated from the semblance as well by replacing ENCCS with semblance, allowing us to generalize the covariance measure function to semblance as well as ENCCS. Obviously, semblance is the most economical.

g)- Both ENCCS and semblance calculate signal and energy at each scan. As long as we assume that there is one event at each scan trajectory and devise our formula accordingly(Eq.10 or 12) there is no extra benefit derived from signal space related concepts (eigenvalue technique) such as dynamically calculating the signal and noise energy. SNR derived from semblance or ENCCS already does this.

Before getting into more details, an introductory review and some definitions are in order.

REVIEW

Gated Velocity Analysis Techniques

The use of temporal gates around the time of interest increases the reliability of the attributes derived from data. All of the references cited above use such gates. During the analysis, a subset of data around the Dix hyperbola

$$t_x = \sqrt{t_0^2 + \frac{x^2}{v^2}} \quad (1)$$

is taken from the data matrix and is analyzed. Gate center times scan the time axis with a time increment which is usually half the gate length. Velocities scan the velocity range with constant or varying increments. For improved resolution, small increments of gate center and velocity are necessary. Picking a gate of data from digital samples requires interpolation. Nearest sample picking, linear interpolation, quadratic interpolation or

static shift techniques such as Lagrange interpolation can be used. Quadratic interpolation is used in the examples given in this paper. The data matrix has N_t rows and N_x columns where N_t is the number of samples in time direction and N_x is the number of traces. The data matrix (a_{ij}) is used to derive a single attribute such as semblance. Afterwards, the attribute is assigned to the attribute array $A(t,v)$, at gate center time t and the scan velocity.

Semblance

Semblance s is the ratio of output energy (the variance of the stack) to the input energy (variance of the CDP) can be expressed as :

$$s = \frac{\sigma_{stk}^2}{\sigma_{CDP}^2} = \frac{\frac{1}{N_t} \sum_{i=1}^{N_t} w_i^2}{\frac{1}{N_t N_x} \sum_{i=1}^{N_t} \sum_{j=1}^{N_x} a_{ij}^2} \quad (2)$$

where w_i is the amplitude of the i -th time sample of the stack wavelet (mean amplitude along offset direction). When signal is uniform along the offset direction and the noise is uncorrelated from trace to trace the signal to noise energy (or variance) ratio, SNR_E can be obtained from semblance using

(3)

$$SNR_E = \frac{s}{1-s}$$

The square root of SNR_E is the signal to noise RMS amplitude ratio (SNR_{rms}).

Covariance Matrix and the energy normalized cross correlation sum

Covariance matrix $\Phi = (\phi_{ij})$ is made of the zero lag values of the unnormalized cross correlations between various gates, (trace i and trace j):

$$\phi_{ij} = \frac{1}{N_t} \sum_{k=1}^{N_t} a_{ki} a_{kj} \quad (4)$$

The dimensions of the covariance matrix are equal to fold of the CDP, N_x . Note that each off-diagonal element in the matrix is a measure of the signal energy and a sizable time gate ($N_t \gg 1$) is needed to statistically stabilize the estimates. For small signal to noise ratio, and for band limited noise, the noise may correlate with signal, giving inconsistent off-diagonal elements in the matrix. Therefore, a further stabilization of the signal estimate is obtained by taking the average of all off-diagonal elements in the matrix:

$$C = \frac{1}{N_x(N_x-1)} \sum_{i \neq j} \phi_{ij} \quad (5)$$

The elements in the main diagonal of the covariance matrix are the auto correlation zero lag values and show the total energy (signal plus noise) in each trace of the data matrix. In the case where the total energy in each trace is approximately equal, then a better estimate of it is done by averaging the elements in the main diagonal:

$$A = \frac{1}{N_x} \sum_{i=1}^{N_x} \phi_{ii} \quad (6)$$

Therefore, in the notation of this paper, the energy normalized cross correlation sum of Neidell and Taner(1971) becomes

$$c = \frac{C}{A} \quad (7)$$

Covariance Measure

The basic assumption Key and Smithson (1990) make is that there is only one event per scan(one event per data matrix). Even if there are indeed more than one event, at the velocity of one of the events, all others are incorrectly moved-out and appear as noise. They define covariance measure from the eigenvalues of the covariance matrix defined above. Therefore for the correct velocity there is always one major eigenvalue, even if there are more than one event. Tying the attribute to the existence of an outstanding major

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

eigenvalue helps and covariance measure is defined as the product of two terms:

$$CM = SNR_E W \quad (8)$$

where SNR_E is the signal to noise (energy) ratio defined from the eigenvalues, and W is the weight factor related to the inequality of the eigenvalues. Noise variance is defined as the mean of all the eigenvalues except the major one:

$$\sigma_n^2 = \frac{1}{N_x - 1} \sum_{i=2}^{N_x} \lambda_i \quad (9)$$

where N_x is the number of traces in the analysis. The largest eigenvalue is considered by Key and Smithson(1990) to be the sum of the signal and noise variance

$$\lambda_1 = \sigma_s^2 + \sigma_n^2 \quad (10)$$

Therefore SNR_E is defined (Key and Smithson, 1990) as

$$SNR_E = \frac{\lambda_1 - \sigma_n^2}{\sigma_n^2} \quad (11)$$

Although Eq.10 and 11 seem to be correct for poor SNR case, they give larger values than SNR_E obtained from Eq. 3. Correct definitions are given by Equations (12) and (13) which should replace Eq.10 and Eq.11

$$\lambda_1 = N_x \sigma_s^2 + \sigma_n^2 \quad (12)$$

$$SNR_E = \frac{(\lambda_1 - \sigma_n^2) / N_x}{\sigma_n^2} \quad (13)$$

This is because the major eigenvalue of an all ones matrix with dimension N_x is not equal to one but equal to N_x .

The weighting coefficient W is defined as the N_x th power of the natural logarithm of the ratio of the arithmetical mean to the geometrical mean

$$W = \rho^{N_x} \quad (14)$$

with

$$\rho = \ln\left(\frac{a}{g}\right) \quad (15)$$

where the arithmetical mean a is given by

$$a = \frac{1}{N_x} \sum_{i=1}^{N_x} \lambda_i \quad (16)$$

and the geometrical mean g is given by

$$g = \left(\prod_{i=1}^{N_x} \lambda_i\right)^{\frac{1}{N_x}} \quad (17)$$

Key and Smithson use a partial stacking scheme and so reduce the fold of the CDP (N_x) by some factor, for example 6. Note that raising ρ to a large power, like 48, would give serious numerical problems, if the partial stacking route is not chosen. Similarly, the geometrical mean of 48 numbers is likely to give numerical problems.

TEST RECORDS

To make the comparison easy with Key and Smithson's results, I have created my synthetics with the same geometry:

Figure 1-A is a 48 trace record at 4 ms sample rate, with one event at 2.000 second with velocity 2700 m/s. Offset increment is 30 m, offsets range from 30 m to 1440 m. The event has a band limited (10-60 Hz) zero phase wavelet of 160 ms duration (cosine taper is 80 ms on each side). The record contains a very small amount of band limited noise (10-60 Hz) (SNR_E calculated over a gate length of 44 ms around the main peak of the wavelet is approximately 1600) .

A velocity increment of 5 m/s is chosen to obtain the velocity spectra. Gate increment is 4 ms and gate length is 44 ms. Figure 1-B presents contour plots for six attributes, four of which (semblance, ENCCS, SNR obtained from semblance, and CM from eigenvalues) have already been discussed. The other two attributes are the extension of CM to semblance and ENCCS and will be discussed in the following sections. Partial

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

stacking is used for CM_{eig} (eigenvalue related covariance measure calculation), with 6 consecutive traces being summed into 1 to reduce fold from 48 to 8 during the calculation of CM_{eig} .

One observes from Figure 1-B the following:

- semblance and ENCCS are virtually identical,
- CM_{eig} has more resolution than semblance as Key and Smithson observed
- SNR obtained from semblance has comparable resolution to CM_{eig}
- All of the attributes show a ridge .

Figure 2-A has the same amount of noise as Figure 1-A but has another event, at 2.012 s with velocity 2725 m/s. Since events are 12 ms apart they are barely separated at small offsets, but indistinguishable at large offsets. The same six attributes are plotted in Figure 2-B.

One can make the following additional observations on this figure

- Interaction of two events distorts the nice ridge that one observes with one event

- It is difficult from semblance contours to assess that there are two events

- SNR_E calculated from semblance brings about enough resolution to predict that there are at least two events but event times and velocities are distorted. We observe that CM_{eig} is more resolved than SNR_E and vividly illustrates the temporal and velocity distortion created due to lack of noise and the size of the time window(44 ms). The two events appear to be pushing each other. This is indeed due to the fact that as 44 ms gate slide along time axis the side lobes of the 2.0 s event that arrive earlier than 2.0 s and sidelobes of the 2.012 s event which arrive later than 2.012 s have higher coherency values than when the gate center is exactly in between the events(2.006 s). We will see below that when data noise increases this distortion disappears. We also observed that (not shown here) as time gate length decreases the distortion observed for SNR = 1600 case gets less and less.

Figure 3-A contains the same single event of Figure 1-A but at SNR_E = 1.0. The same six attributes are shown in Figure 3-B. One can observe the following

- Semblance and ENCCS are about the same, ENCCS being a little cleaner,
- SNR_E obtained from semblance is slightly more

resolved than semblance and ENCCS,

- CM obtained from eigenvalues has more resolution than semblance, ENCCS, and SNR_E obtained from semblance,

- The "Ridge" observed in Figure 1-B is now smaller in temporal extent.

Figure 4-A is the two event record of Figure 2-A but at SNR_E = 1.0. It is not possible to tell visually that there are two events. Figure 4-B presents the attributes.

We observe that

- Although there is a saddle point in semblance it is hard to detect it from the contour plot. ENCCS is similar to semblance. Compared to a single event in Figure 3-B, contours are more circular (contours look more like bull's eye).

- SNR obtained from semblance does not help resolution.

- Covariance measure obtained from eigenvalues show only one event. This is due to particular noise distribution and the 44 ms long temporal gate. For smaller time gates like 20 ms (not shown here) we were able to resolve the two events.

SIGNAL TO NOISE RATIO VERSUS COVARIANCE MEASURE

The simulation experiments performed (although not shown here) which compare SNR_E obtained from eigenvalues to SNR_E obtained from semblance show that they are same after the correction done by Eq. 12 and 13. These experiments also show(not shown here) that W factor has less resolution than SNR when signal is good, but has more resolution than SNR_E when signal-to-noise ratio is poor. Also, in the latter case, the results are erratic. It is already observed above (Figure 1-B) that for high signal-to-noise ratio, the SNR_E obtained from semblance has a similar resolution to CM_{eig} . Therefore one might be inclined to use SNR_E obtained from semblance, and ignore W. The reason why SNR_E (obtained from semblance) is so sharp for good signal-to-noise ratio is the fact that as semblance s in Eq.3 approaches unity, we get

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

$$SNR_E = \frac{1}{\mu} \quad (18)$$

where

$$\mu = 1 - \epsilon \quad (19)$$

is a small number.

However, the comparison of SNR_E obtained from semblance to CM_{Eq} for various noise levels (Figure 3-B, 4-B and 5-B) show that when signal-to-noise ratio is poor, CM_{Eq} is more resolved. Therefore the resolution must be coming from W . However, it has been observed that many highly resolved but inaccurate CM_{Eq} spectra may result for small signal-to-noise ratio. This is mostly due to exponent N_x in Eq. 14 and using small amounts of processing white noise (to be discussed later).

It is observed above (Figure 3-B and 4-B) that for poor signal-to-noise ratio, SNR_E obtained from semblance has no improvement over semblance. This is not surprising since SNR_E given in Eq. 3 approaches semblance as semblance goes to zero. However, raising SNR_E or semblance to a power would bring back the resolution (along with the danger of boosting bogus events). It will be shown in a later section that weight W is doing exactly that (raising semblance to a power) when SNR_E is smaller than one.

COVARIANCE MEASURE FOR HIGH SNR AND PROCESSING WHITE NOISE

It has been mentioned above that for a single event, signal trajectory gives a covariance matrix which is made of a constant:

$$\Phi = \sigma_s^2 \begin{pmatrix} 1, & 1, & \dots, & 1 \\ 1, & 1, & \dots, & 1 \\ \dots, & \dots, & \dots, & \dots \\ 1, & 1, & 1, & \dots, & 1 \end{pmatrix} \quad (20)$$

The rank of the matrix is one (it is singular), and all its eigenvalues except one are zero. Such a matrix may create problems for some eigenvalue solvers. Furthermore, geometric mean calculation gets into

trouble. To eliminate such problems, one can add a small white noise into the main diagonal of the covariance matrix:

$$\Phi = \sigma_s^2 \begin{pmatrix} 1+\epsilon, & 1, & \dots, & 1 \\ 1, & 1+\epsilon, & \dots, & 1 \\ \dots, & \dots, & \dots, & \dots \\ 1, & 1, & 1, & \dots, & 1+\epsilon \end{pmatrix} \quad (21)$$

where ϵ is a small positive number. Then, the eigenvalues of the matrix change from

$$N_x \sigma_s^2, 0, 0, \dots, 0 \quad (22)$$

to

$$N_x \sigma_s^2 (1+\epsilon), \epsilon, \epsilon, \dots, \epsilon \quad (23)$$

Then SNR given in Eq. 13 becomes

$$SNR_E \approx \frac{1}{\epsilon} \quad (24)$$

The arithmetical mean approaches to

$$a \approx \sigma_s^2 \quad (25)$$

and the geometrical mean approaches to

$$g \approx \epsilon \sigma_s^2 \quad (26)$$

therefore, the log-generalized likelihood ratio becomes

$$\rho \approx \ln\left(\frac{1}{\epsilon}\right) \quad (27)$$

Note that ϵ can be viewed as the noise that is contained in data for high signal-to-noise ratio case. Comparing Eq. 18, 19, and 24 implies that ϵ acts like $\mu = 1 - \text{semblance}$. Therefore, sensitivity of CM to ϵ also determines the sensitivity to random noise in data. Since most sensitivity to c is in Eq. 24 but not in Eq. 27, this explains why SNR_E term has the same resolution with CM for high signal-to-noise ratio data. The examples of CM_{Eq} given above are run with 0.1 percent white noise. By making white noise less, for

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

-example 0.0001 percent, increases the resolution, yet increases the erratic nature of the results for poor data

GENERALIZATION OF COVARIANCE MEASURE TO ENCCS AND TO SEMBLANCE

Despite the stability obtained from the addition of white noise to the main diagonal of the covariance matrix, spurious peaks was obtained in CM_{sig} when signal to noise ratio was poor. The study of the covariance matrices for such records revealed two facts : a)-there are many negative elements in the covariance matrix due to random noise or incorrect scan velocity , b)-spurious peaks occur when there are very small eigenvalues. Since, for trajectories aligning the signal there should not be any negative element in the covariance matrix, one might choose to zero the negative elements before eigenvalue calculations (due to one event assumption, this is not harmful). Also, one may limit the lowest value of the eigenvalues by an amount controlled by white noise ϵ . That is, any eigenvalue which falls below

$$\lambda_{\min} = \epsilon \lambda_{\max} \quad (28)$$

is set to that value. The reason for this is that even for pure noise, the minimum eigenvalue is equal to the variance of the least noisy trace. These two modifications resulted in significant improvements (not shown here) in the CM runs, which prompted the following question: what if we replaced every off-diagonal element in the covariance matrix with their mean value C and every element on the main diagonal with their mean A. When this is done one obtains :

$$\Phi = \begin{pmatrix} A & C & C & \dots & C \\ C & A & C & \dots & C \\ C & C & A & \dots & C \\ \dots & \dots & \dots & \dots & \dots \\ C & C & C & \dots & A \end{pmatrix} \quad (29)$$

A few calculations with such a matrix shows that the eigenvalues of the matrix are

$$\lambda_1 = A - C + N_x C, \quad \lambda_2 = \lambda_3 = \dots = \lambda_{N_x} = A - C \quad (30)$$

Note that noise variance given by Eq. 9 becomes $\sigma_n^2 = A - C$ and signal variance given by Eq. 12 is $\sigma_s^2 = C$, and the SNR_E given by Eq. 13 becomes

$$SNR_E = \frac{C}{1 - C} \quad (31)$$

where c is defined by Eq. 7. Therefore Eq.31 can be used to define SNR_E for ENCCS and is parallel to Eq. 3 used for semblance.

The arithmetical mean of the eigenvalues is A and the geometrical mean of the eigenvalues are

$$g = \left((A - C)^{N_x - 1} (A - C + N_x C) \right)^{\frac{1}{N_x}} \quad (32)$$

Therefore the ratio of the geometrical mean to the arithmetical mean will be

$$\frac{g}{A} = \frac{1}{\left((1 - C)^{N_x - 1} (1 + (N_x - 1) C) \right)^{\frac{1}{N_x}}} \quad (33)$$

Noting that this ratio goes to infinity for good signal (c = 1) and goes to one when there is no signal (c = 0), and the fact that this factor is needed for velocity resolution for poor data, one can approximate it with 1/(1-c) and therefore the log-generalized likelihood ratio defined in Eq. 13 becomes

$$\rho = \ln\left(\frac{g}{A}\right) = \ln\left(\frac{1}{1 - C}\right) \quad (34)$$

Since semblance and ENCCS are very close to each other, the log-generalized ratio corresponding to semblance would be obtained from Eq. 34 by replacing c with semblance s:

$$\rho = \ln\left(\frac{1}{1 - S}\right) \quad (35)$$

Therefore, covariance measure can be defined for all three cases, the one obtained from eigenvalues, the one

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

obtained from ENCCS, and the one obtained from semblance as

$$CM = (\text{attrib})^{\rho} (SNR_E)^{\beta} (\rho)^{\gamma} \quad (36)$$

Here, SNR_E is defined by Eq. 3 for semblance, Eq. 24 for ENCCS, and Eq. 13 for eigenvalues. ρ in this expression is defined by Eq. 35 for semblance, Eq. 34 for ENCCS and Eq. 15, 16, 17 for the eigenvalue technique. Variable "Attrib" is either semblance (s) or ENCCS (c) or, in the case of the eigenvalue technique, is obtained from SNR_E using

$$\text{Attrib} = \frac{SNR_E}{1 + SNR_E} \quad (37)$$

Introduction of "Attrib" into Eq. 36 allows one the flexibility to use a raw attribute such as semblance by setting powers of SNR and ρ to zero.

CM obtained from ENCCS (CM_{cc}) is shown in Figures 1-B, 2-B, 3-B, 4-B along with CM obtained from eigenvalues (CM_{eig}). It is observed that CM_{cc} is more stable than CM_{eig} at all noise levels. It can also be observed that CM obtained from semblance (CM_{sem}) is almost identical to CM_{cc} . Since semblance is inexpensive to calculate this gives us a tool as powerful as covariance measure with much less cost.

CONCLUSION

The SNR_E obtained from semblance has similar resolution to S/N defined by Key and Smithson for good data. For poor data, resolution of the covariance measure is better than that of SNR and comes from the weight factor. These observations and theoretical considerations given in the text lead to the generalization of the covariance measure concept to semblance as well as to energy normalized cross correlation sum. Covariance measures obtained from semblance and ENCCS have the same resolution with the covariance measure obtained from the eigenvalues, at all noise levels of the data. The CM originating from semblance is more robust and much cheaper to calculate than the CM originating from the eigenvalues.

Regardless of the origin of calculation, covariance measure lacks accuracy in both velocity and time, despite its sharp response for noise free case. This

inaccuracy is dependent on the gate length, the duration of the wavelet residing on the events, the amount of random noise present on the data and the amount of white noise added to the main diagonal of the covariance matrix. The lesser the amount of random noise in data, the bigger the errors. This behavior is the result of using gates and is common to all gated velocity analysis techniques.

ACKNOWLEDGMENTS

I thank Cam Wason for suggesting the study and for various discussions during its progress. I also thank Dave Monk for helpful suggestions. I am grateful to Halliburton Geophysical Services for allowing me to publish this work.

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310

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

FIGURE 1-A

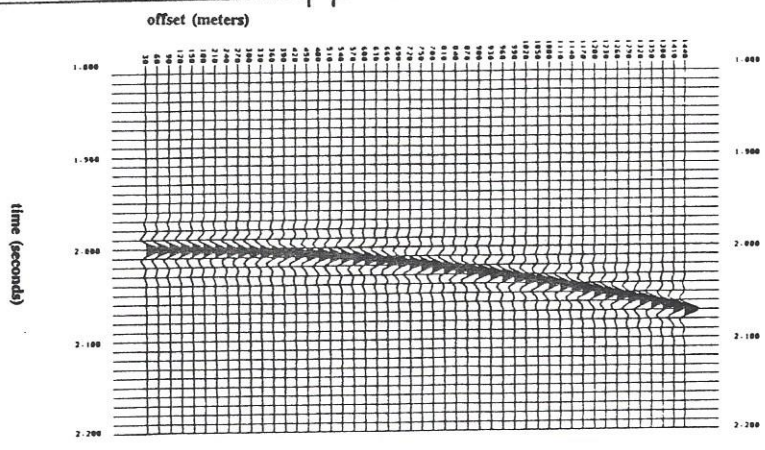
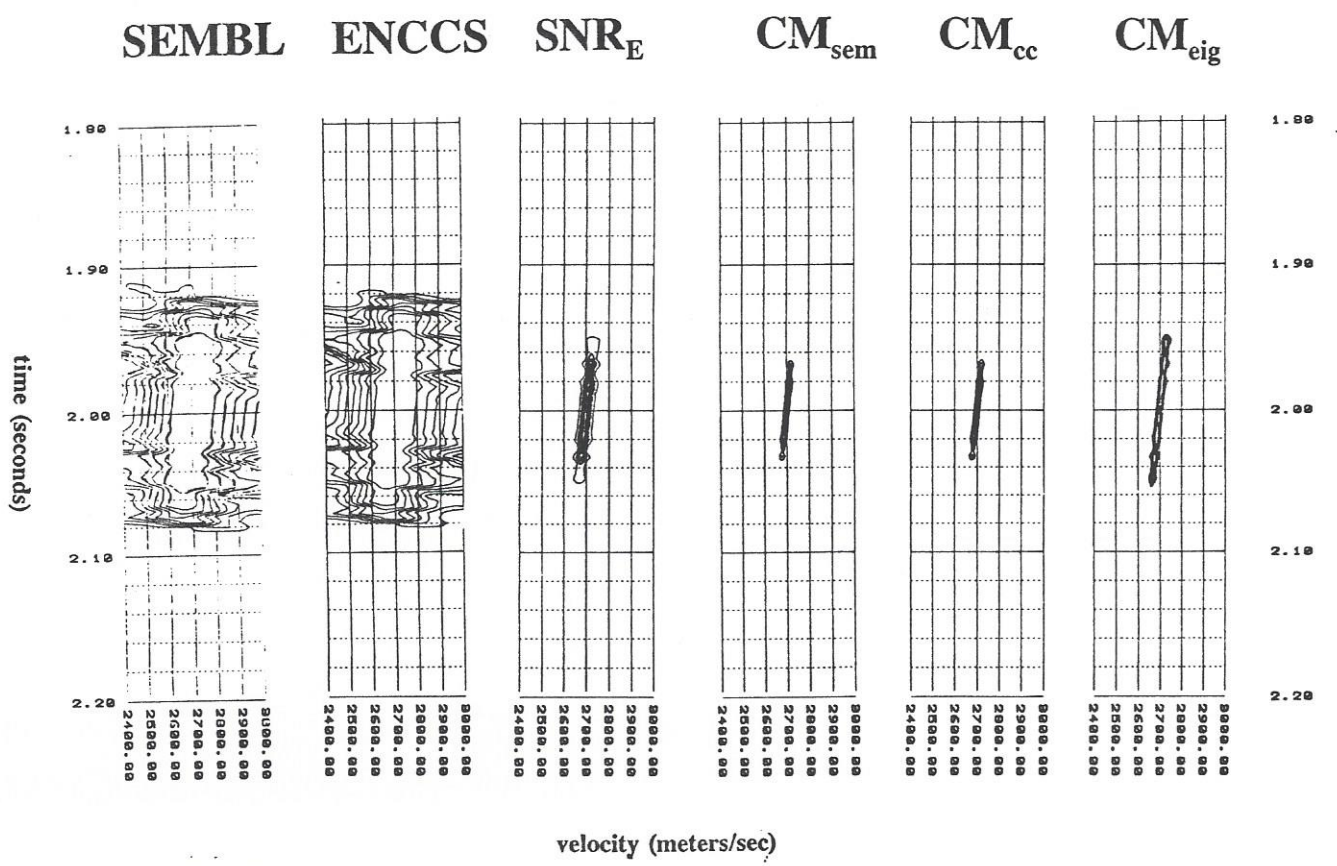


FIGURE 1-B



ONE EVENT, $V = 2700$ m/s, $t_0 = 2.000$ s, $SNR_{44ms} = 1600$, WAVELET = 160 ms, GATE = 44 ms

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

FIGURE 2-A

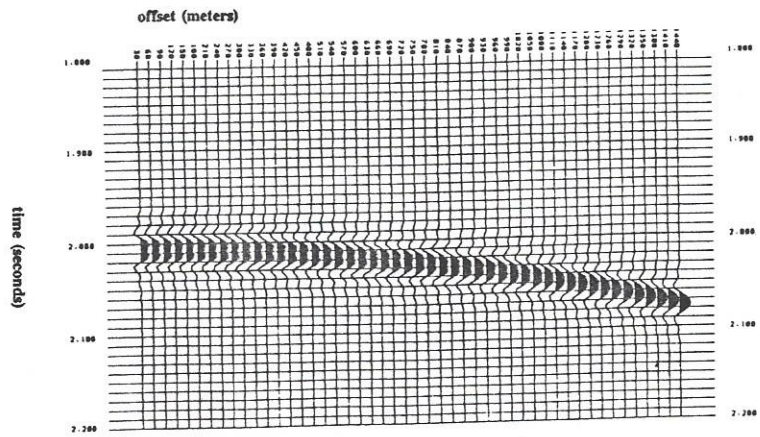
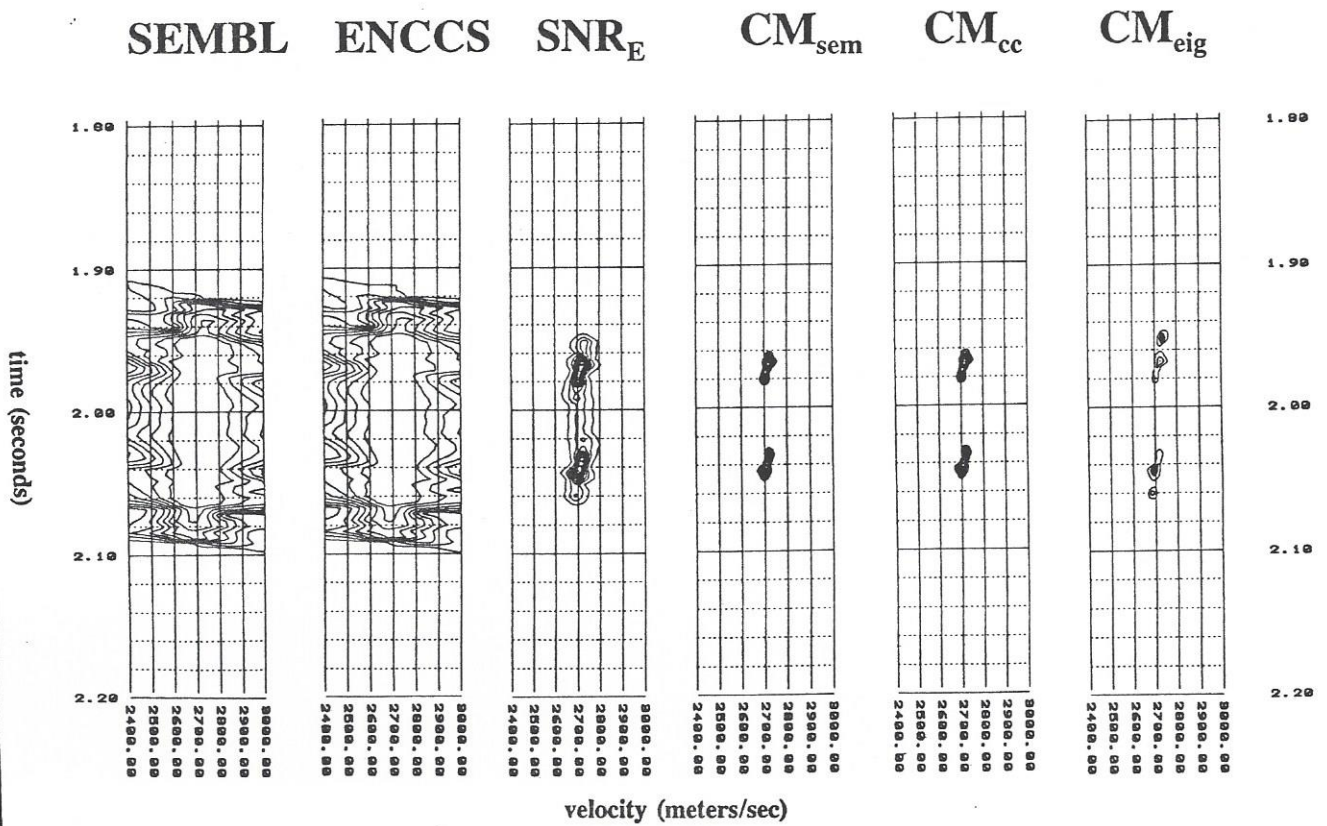


FIGURE 2-B



TWO EVENTS, $V=2700, 2725$ m/s, $t_0=2.000, 2.012$ s, $SNR_{44ms}=1600$, WAVELET=160 ms, GATE=44 ms

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

FIGURE 3-A

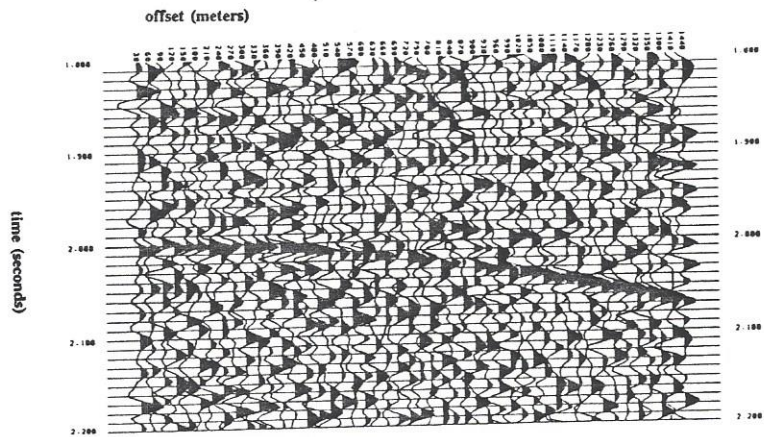
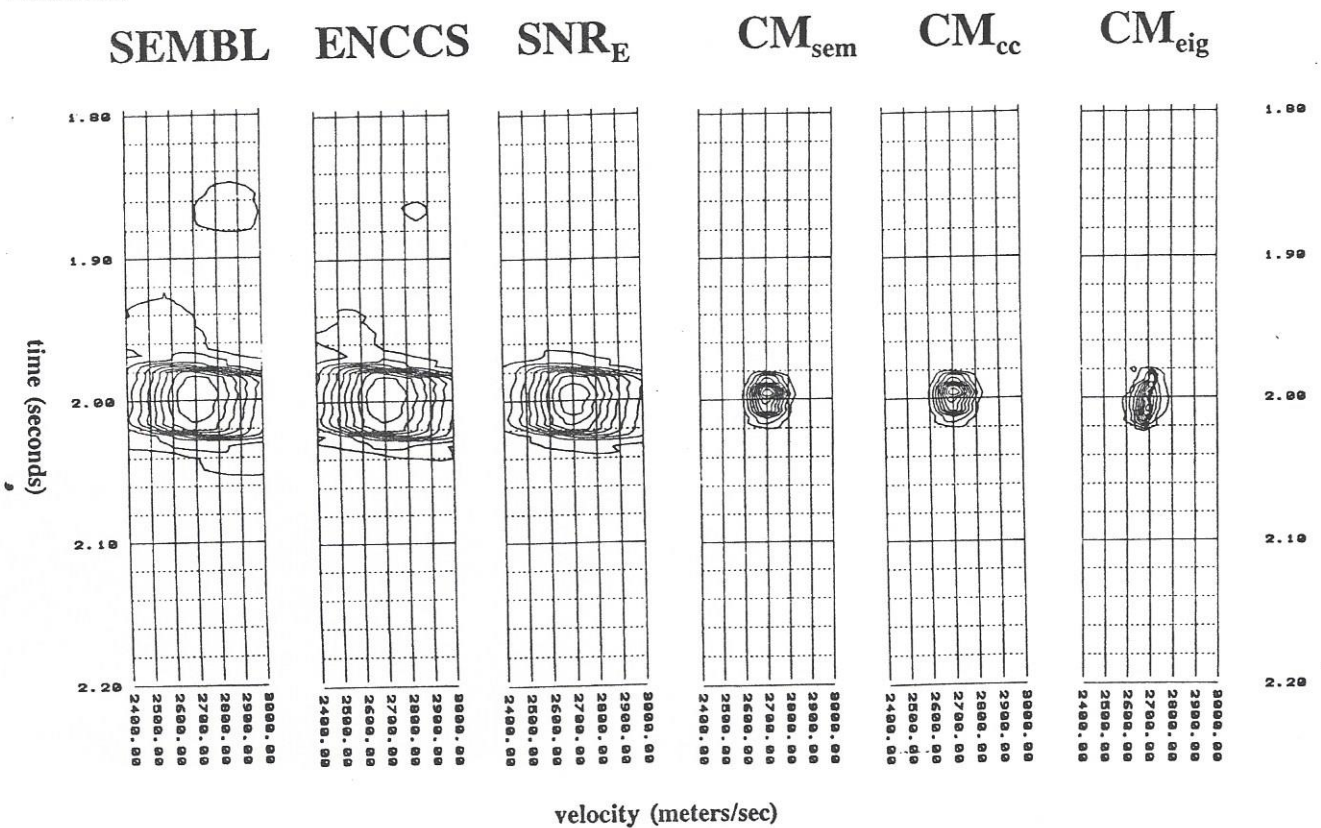


FIGURE 3-B



ONE EVENT, V=2700 m/s, $t_0=2.000$ s, $SNR_{44ms}=1$, WAVELET=160 ms, GATE=44 ms

COVARIANCE MEASURE AS A HIGH RESOLUTION VELOCITY ANALYSIS TOOL

KOVARYANS ÖLÇÜMÜNÜN HIZ AYRIM GÜCÜ YÜKSEKTİR

FIGURE 4-A

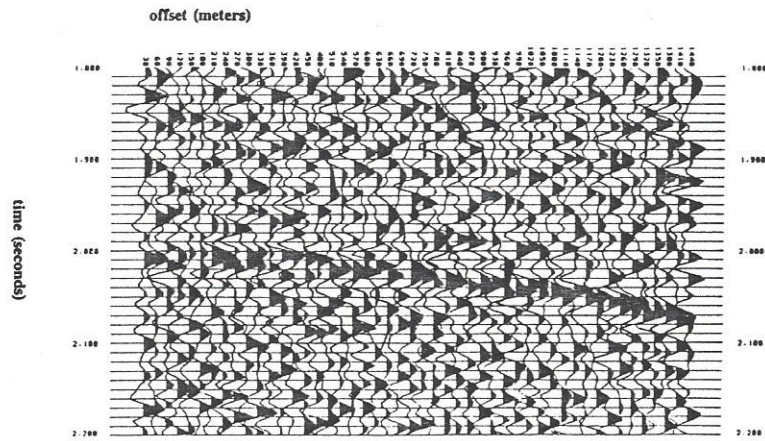
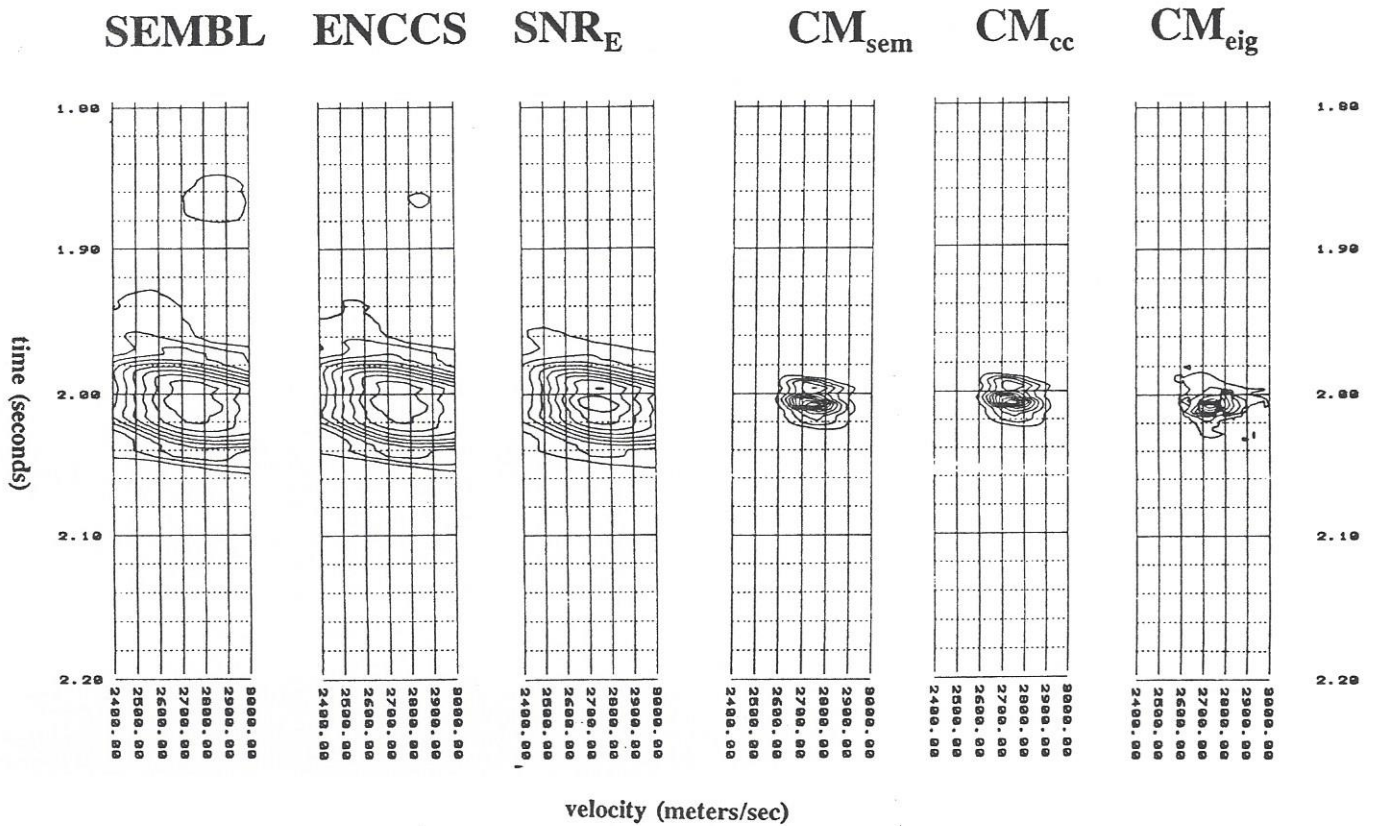


FIGURE 4-B



TWO EVENTS, $V=2700, 2725$ m/s, $t_0=2.000, 2.012$ s, $SNR_{44ms}=1$, WAVELET=160 ms, GATE=44 ms