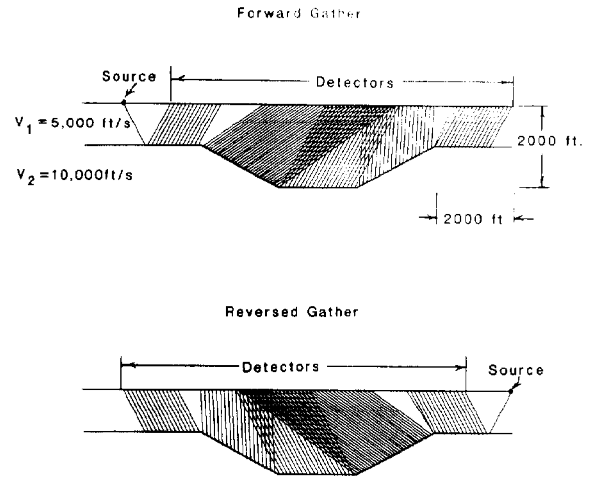

FIG. 1. A forward and reversed ray trace through a model of a graben filled with sediment.

but they serve to demonstrate the downward continuation method. It would be difficult to interpret the first arrivals on these seismograms because of the shadow zones and superimposed arrivals. Downward continuation can help solve this problem.

The numerical downward continuation is done for the forward and reversed data by a summation method. The wave field at each point in space is constructed by summing traces recorded by geophones within a chosen aperture. Each trace in the sum is shifted by the delay time between the geophone and the spatial point. The sum of the traces is then phase shifted by 45 degrees. Figure 3 is the image that comes from taking products of the forward and reversed fields in the manner described above. The "first break" in depth gives a good delineation of the boundary. The downward continuation resulted in a simplification of the interpretation of the first arrivals for this synthetic example. The diffractions in the image are a consequence of the ray trace synthetic and spatial truncation of the data. When an interface has very high relief, the shortest traveltime path may differ significantly from a path along the interface. This difference causes some error in the method (Rockwell, 1967).

The method could be practical for interpreting real data, especially in areas where shadow zones and diffractions make cycle skipping difficult to avoid. The images formed from different pairs of forward and reversed gathers could be stacked. In many

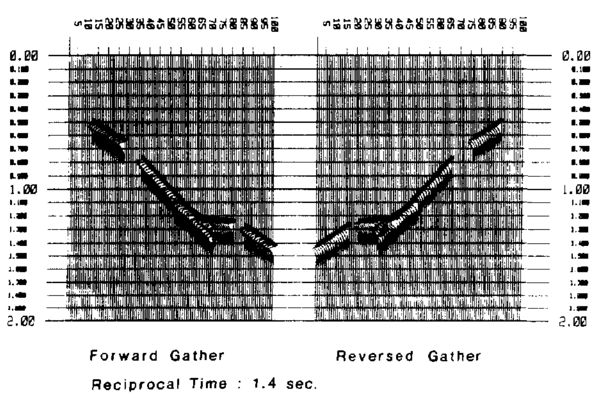

FIG. 2. Ray trace synthetic seismograms for forward and reversed directions. There are shadow zones and superimposed arrivals.

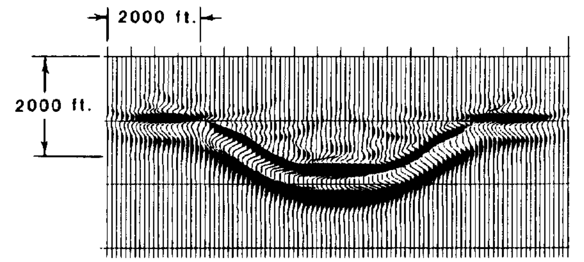

FIG. 3. Image of refracting interface obtained by downward continuation of refracted arrivals of forward and reversed gathers.

locations, the success of the method probably will depend on incorporating a technique for overcoming residual statics caused by local delays associated with shot and receiver positions.

This paper has considered only a two-layer case. Simple models such as these are adequate in many locations and are often used when interpreting refraction arrivals (Jones and Jovanovich, 1985). No assumption was made about the refractor velocity v_2. The downward continuation used only the overburden velocity v_1. When several layers exist, the present method might be extended, as has been done with the graphical methods. Apparently, we could only delineate one interface at a time, using the reciprocal time appropriate for the refraction from the interface being considered.

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A New Method for the Surface-Consistent Decomposition of Statics Using Diminishing Residual Matrices (DRM) S1.3

Necati Gulunay, Petty-Ray Geophysical

Whether it is a refraction or reflection statics problem, one always encounters the problem of decomposing matrices into certain components of which the matrix is a sum. Gauss-Seidal algorithm is one such algorithm which decomposes matrices into certain components (shot term, receiver term, structure term, and offset term). Here we offer a different method, the diminishing residual matrix method (DRM) for cases where the static matrix is assumed to be the sum of the two terms only (shot and receiver terms).

In refraction statics, the matrix to be decomposed is the linear moved out first break arrival times, i.e., total delay times are the instantaneous intercept times. The problem then is to decompose total delay times surface consistently into shot and receiver delay times. We applied the DRM method to this problem and were quite successful. Indeed, experience with the application of DRM to refraction statics has shown that not only can we solve for high frequency statics, but also for very long wavelengths compared to spread length.

By explaining the method in simple terms, we will try to show here why it works.

Introduction

Most recent reflection static programs try to decompose a $\mathbf{T}_{ijk}$ matrix:

$$\mathbf{T}_{ijk} = S_j + R_i + G_k + X_k \qquad k = \frac{i+j}{2},$$

into its shot (S_j), receiver (R_i), structure (G_k), and offset (X_k) components (Taner et al., 1971; Wiggins et al., 1976). If one can assume that offset dependence is negligible and net CDP shifts are products of coinciding and large shot-receiver profiles above those CDPs, then $\mathbf{T}_{ijk}$ measurements can be averaged over a CDP or a number of CDPs and by subtracting this average value at the CDP for all elements of $\mathbf{T}_{ijk}$ at that CDP one gets:

$$\mathbf{T}_{ij} = S_j + R_i,$$

where $\mathbf{T}_{ij}$ are total residual times (no structure involved unless smoothing kept too long). One comes to the same conclusion if one uses a pilot trace for a CDP (designed at or around that CDP, yet centered at that CDP) and obtains residual times $\mathbf{T}_{ij}$ by correlating components of the CDP to this pilot (Hileman et al., 1968). Therefore, the problem here, given $\mathbf{T}_{ij}$ matrix, is to find shot array S_j and receiver array R_i.

Similarly, in refraction statics traveltime $\mathbf{T}'_{ij}$ from shot j and receiver i reduces to total travel time $\mathbf{T}_{ij}$ after linear moveout at the refractor velocity:

$$\mathbf{T}_{ij} = \mathbf{T}'_{ij} - \frac{X_{ij}}{V} \qquad \text{(constant refractor velocity V)}$$

$$\mathbf{T}_{ij} = \mathbf{T}'_{ij} - \frac{X_{ij}}{i-j}\sum_{m=1}^{j-i}\frac{1}{V(i+m)} \text{(changing refractor velocity and } j > i\text{)},$$

$$\mathbf{T}'_{ij} - \frac{X_{ij}}{j-i}\sum_{m=1}^{i-j}\frac{1}{V(i-m)} \quad \text{when } i > j,$$

where X_{ij} is the distance between jth shot and ith receiver. We assume here that refractor velocity profile is known.

Therefore, one is left with delay time matrix $\mathbf{T}_{ij}$ to be decomposed into shot delay S_j and receiver delay R_i

$$\mathbf{T}_{ij} = S_j + R_i.$$

The problem then is to find a method to accomplish this.

The Gauss-Seidal iterative solution is one such method (Wiggins et al., 1976; Taner et al., 1974; Moser and Jovanovich, 1984; Farrel and Euwema, 1984). However, the Gauss-Seidal method is not always convergent (Kellison, 1975) and to create a diagonal heavy matrix that insures convergence, one has to inject arbitrary unknowns similar to white noise in deconvolution.

Here, we describe a new method (DRM) accurately solves for

$$\mathbf{T}_{ij} = \mathrm{S}_j + \mathrm{R}_i$$

even for long wavelengths equal to line length. During the explanation of the method, we also show that the DRM method is equivalent to the least-squares error method for square matrices.

Diminishing residual matrices method (DRM)

Let $\mathcal{R}_{ij,1} = \mathbf{T}_{ij}$ be the original observation matrix that may have only some elements known. Define the residual matrix at $k + 1$th iterations as:

$$\mathcal{R}_{ij,k+1} = \mathcal{R}_{ij,k} - r_{i,k} - s_{j,k},$$

where

$$r_{i,k} = \frac{1}{2F_i^R}\cdot\sum_{j=1}^{F_i^R} \mathcal{R}_{ij,k} \text{ is the residual receiver static at } k\text{th iteration;}$$

$$s_{j,k} = \frac{1}{2F_j^S}\cdot\sum_{i=1}^{F_j^S} \mathcal{R}_{ij,k} \text{ is the residual shot static at } k\text{th iteration,}$$

and F_i^R is the fold for the ith receiver (number of shots contributing to the calculation of ith receiver) and F_j^S is the fold for the jth shot (number of receivers contributing to the static calculation). Then, the total static at ith receiver and jth shot are

$$r_i = \sum_{k=1}^{N} r_{i,k} \qquad s_j = \sum_{k=1}^{N} s_{j,k},$$

where N is the number of iterations.

Example 3 × 3 Case: Solution with least-square difference constraint

$$\begin{bmatrix} s_1+r_1 & s_2+r_1 & s_3+r_1 \\ s_1+r_2 & s_2+r_2 & s_3+r_2 \\ s_1+r_3 & s_2+r_3 & s_3+r_3 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}.$$

We have 9 equations and 6 unknowns. If one attempts to solve for the unknowns, one easily discovers that four of the equations are redundant because of the following relationships: $T_{13} + T_{31} = T_{11} + T_{33}$; $T_{23} + T_{31} = T_{21} + T_{33}$; $T_{23} + T_{12} = T_{22} + T_{13}$; $T_{12} + T_{33} = T_{13} + T_{32}$. Therefore, we effectively have 5 equations and 6 unknowns and those equations can be solved only in terms of one of the unknowns. By changing to $d_k = s_k - r_k$ ($k = 1, 2, 3$), the equations are solved in terms of d_3: $r_1 = T_{13} - 0.5T_{33} - 0.5d_3$; $r_2 = T_{23} - 0.5T_{33} - 0.5d_3$; $r_3 = 0.5T_{33} - 0.5d_3$; $d_1 = T_{31} - T_{13} + d_3$; $d_2 = T_{32} - T_{23} + d_3$.

The independent variable d_3 can be obtained if one imposes minimum power condition; $P = d_1^2 + d_2^3 + d_3^2$, $dp/dd_3 = 0$. This yields $d_3 = ((T_{13} - T_{31}) + (T_{23} - T_{32}))/3$. As an example, if we were given:

$$T = \begin{bmatrix} 8 & 16 & 21 \\ 13 & 21 & 26 \\ 18 & 26 & 31 \end{bmatrix},$$

then the above equations would give $d_3 = 1$, $r_1 = 5$, $r_2 = 10$, $r_3 = 15$, $d_1 = -2$, $d_2 = 1$, therefore $s_1 = 3$, $s_2 = 11$, $s_3 = 16$.

3 × 3 Case: Solution with DRM method.

Let us give the solution for the example above but using DRM:

$$T = \begin{bmatrix} 8 & 16 & 21 \\ 13 & 21 & 26 \\ 18 & 26 & 31 \end{bmatrix} = \begin{bmatrix} r_{1,0} \\ r_{2,0} \\ r_{3,0} \end{bmatrix} = \begin{bmatrix} 7.5 \\ 10.0 \\ 12.5 \end{bmatrix}\begin{bmatrix} s_{1,0} \\ s_{2,0} \\ s_{3,0} \end{bmatrix} = \begin{bmatrix} 6.5 \\ 10.5 \\ 13.0 \end{bmatrix}.$$

The expected static from the solution and residual matrix is

$$\mathscr{E}_1 = \begin{bmatrix} 14.0 & 18.0 & 20.5 \\ 16.5 & 20.5 & 23.0 \\ 19.0 & 23.0 & 25.5 \end{bmatrix} \mathcal{R}_1 \mathrm{T} - \mathscr{E}_1$$

$$= \begin{bmatrix} -6.0 & -2.0 & 0.5 \\ -3.5 & 0.5 & 3.0 \\ -1.0 & 3.0 & 5.5 \end{bmatrix}.$$

If one iterates one more:

$$\begin{bmatrix} r_{1,1} \\ r_{2,1} \\ r_{3,1} \end{bmatrix} = \begin{bmatrix} -1.25 \\ 0 \\ 1.25 \end{bmatrix}\begin{bmatrix} s_{1,1} \\ s_{2,1} \\ s_{3,1} \end{bmatrix} = \begin{bmatrix} -1.75 \\ 0.25 \\ 1.50 \end{bmatrix}$$

$$\mathscr{E}_2 = \begin{bmatrix} -3.00 & -1.00 & 0.25 \\ -1.75 & 0.25 & 1.50 \\ -0.50 & 1.50 & 2.75 \end{bmatrix},$$

and residual matrix $\mathscr{R}_2 = \mathscr{R}_1 - \mathscr{E}_2$

$$= \begin{bmatrix} -3.00 & -1.00 & 0.25 \\ -1.75 & 0.25 & 1.50 \\ -0.50 & 1.50 & 2.75 \end{bmatrix}.$$

Note that the expected matrix $\mathscr{E}_2$ as well as the residual matrix $\mathscr{R}_2$ is half of what we started with:

$$\mathscr{E}_2 = \frac{1}{2}\mathscr{R}_1, \qquad \mathscr{R}_2 = \frac{1}{2}\mathscr{R}_1.$$

Therefore, the solution diminishes by ratio 2 as well. Full solution then is:

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} 7.5 \\ 10.0 \\ 12.5 \end{bmatrix} + \begin{bmatrix} -1.25 \\ 0 \\ 1.25 \end{bmatrix} \underbrace{(1 + \frac{1}{2} + \frac{1}{4} + \ldots)}_{2}$$

$$= \begin{bmatrix} 5 \\ 10 \\ 15 \end{bmatrix},$$

$$\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 6.5 \\ 10.5 \\ 13.0 \end{bmatrix} + \begin{bmatrix} -1.75 \\ 0.25 \\ 1.50 \end{bmatrix} \underbrace{(1 + \frac{1}{2} + \frac{1}{4} + \ldots)}_{2}$$

$$= \begin{bmatrix} 3 \\ 11 \\ 16 \end{bmatrix}.$$

Note that this is the same result as obtained by the least-squares technique above.

What makes the residual matrix diminish by the power of 2 is its properties:

$$R_{11} + R_{22} + R_{33} = 0 \qquad R_{31} + R_{22} + R_{13} = 0$$

$$R_{21} + R_{32} + R_{13} = 0 \qquad R_{21} + R_{12} + R_{33} = 0$$

$$R_{12} + R_{23} + R_{31} = 0 \qquad R_{32} + R_{23} + R_{11} = 0.$$

This also insures that each residual solution has a zero mean.

We observed similar properties in large square matrices also. We can deduce from here that for square matrices (i.e., shots are done at every station and spread is as long as the line), the DRM method provides the identical results one would get from a least squares solution where the power in the difference profile

$$P = (s_1 - r_1)^2 + (s_2 - r_2)^2 + \ldots + (s_n - r_n)^2$$

is minimized.

Numerical tests with nonsquare matrices and matrices that do not have observation from some elements (line is much longer than the spread) gave results in agreement with both the least-squares and the Gauss-Seidal solution, as long as divisions are done by twice the fold for that element, rather than by just the fold.

Since residual solutions diminish by a ratio approximately equal of 2 at each iteration, even the largest static case of refraction statics can be solved in less than 10 iterations (2^{10} = 1 024 ms).

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Refraction Modeling for Static Corrections S1.4

William A. Schneider and Shih-Yen Kuo, Golden Geophysical

The reflection seismic prospecting method has largely ignored refraction arrivals since evolution of the CDP stack and digital processing in the late 1950s and early 1960s. Over the past several years, however, there has been a resurgence of interest in refraction theory and analysis to facilitate near-surface traveltime corrections (statics) and improve the structural integrity of the stacked section with regard to long wavelength statics. A number of commercial programs now exist which routinely incorporate first-break traveltime information to provide a near-surface (one or two layer) model, which can be used for static correction to some reference datum surface. These programs exploit a variety of classical refraction methods, which when coupled with highly redundant refraction observations from conventional CDP coverage, generally yield satisfactory results.

This paper describes a new method for refraction analysis in which first-break observations are inverted to give a near-surface model by the generalized linear inversion (GLI) procedure. The GLI method has found numerous applications in geophysics and appears to be a powerful tool for refraction modeling when the near-surface layering is complex. The GLI method is outlined in the talk and illustrated with field examples comparing this new method with conventional refraction static solutions.

Introduction

The history of reflection seismic processing consists of a series of important breakthroughs and innovations which, at the time of their introduction, represented major advances in the state of the art. Most of these improvements in the reflection seismic method were the result of new concepts or ideas being put into practice (e.g., CDP stack, deconvolution, wave-equation migration, etc.). A few were the result of rediscovering or reengineering ideas and concepts that had been known for many years. Modern refraction statics fall into this category.

Refraction theory and interpretation was a highly developed art when the reflection seismic method was invented in the late 1920s, and many of the early seismic crews successfully employed refraction methods to outline shallow Gulf Coast salt domes. The reflection method, with greater depth of penetration and resolution, quickly displaced refraction techniques as the preferred seismic prospecting tool, although refraction theory and analysis still played an important role with regard to weathering corrections for reflection records. This situation prevailed until the early 1960s when digital seismic recording and processing were introduced. The digital revolution profoundly and irrevocably changed the reflection seismic method. As digital processing