TRACE INTERPOLATION WITH DATA ADAPTIVE FILTERING IN THE FREQUENCY-WAVENUMBER DOMAIN

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SUMMARY

By using a data adaptive filtering method on the zero-trace inserted data one can obtain a generalized trace interpolation. The interpolation filter can be designed and applied in the f-k domain. This method can interpolate 2-D as well as 3-D data efficiently to lessen the artifacts that multichannel processes produce on aliased data.

INTRODUCTION

Trace interpolation for aliased data has received much attention in the last decade. Because aliased data create artifacts for multichannel processes trace interpolation prior to such processes is done to lessen such artifacts. Development and use of trace interpolation methods that make use of the f-k domain has emerged in recent years. This domain is of interest because of the availability of fast Fourier transforms. In this domain interpolation is done with a point-by-point multiplication of the input and an interpolation operator. Using this approach a "generalized f-k trace interpolation"[1,2] method that can interpolate 2-D as well as 3-D data was recently developed. This method can be viewed as a data adaptive t-x domain filtering method designed and applied in the frequency-wavenumber domain. In this paper, we will briefly present the method and discuss in detail filtering aspects of this interpolator. We will also show results on 2-D and 3-D field data.

1-D INTERPOLATION IS INSUFFICIENT

Interpolation schemes for 1-D data are band-limited in the sense that the data's spectrum after interpolation is confined to the range before interpolation. Such an interpolation (when done along the space direction) would be unacceptable if the data contained steeply dipping events. That is, the wrapped f-k spectrum has to be unwrapped by the interpolation process by making use of more than one dimension, and thereby extending the wavenumber spectrum beyond the wavenumber range supported by the recorded data. That is what "generalized f-k trace interpolation" does.

A DATA ADAPTIVE INTERPOLATION FILTER

The basic idea of "generalized f-k trace interpolation" is that, for a given temporal frequency, f, the Fourier transform of interpolated data can be obtained by a point-by-point multiplication on the copied version of the original transform at that frequency. Note that copying of the transform is done along the wavenumber axis. This interpolator is equivalent to a two-dimensional filter in the t-x domain convolving with the zero-trace-inserted data. Assuming the input data are made of linear events, the interpolation filter at a given temporal frequency that will generate a space grid that is L times denser than the original space grid can be obtained from a temporal frequency that is L times lower than the original temporal frequency.

Figures 1 and 2 describe what happens to Fourier transforms for L=2, (L is interpolation factor) when one pads the data with zeroes, inserts zeroes to it periodically, or when one masks zero-padded data with zeroes periodically. For an input of length N along space direction the copied transform which is of length NL corresponds to zero trace-inserted version of the original data (Appendix-1 and Figure 1). The length of the filter along the wavenumber axis is also NL. This filter, H(f,K), is calculated by taking the ratio of the stretched Fourier transform, D(f/L, K/L) and its L times wrapped version along wavenumber axis, D_z(f/L, K/L). The stretch is L-times and is along both frequency and wavenumber axis. Note that stretching (Appendix 2) does not alter the corresponding Nyquist frequencies and it just interpolates transform samples. Note also that

$D_z(f/L,K/L)$ corresponds to the periodically zero masked original data (see lower half of Figure 1 and Appendix 3). It is periodic with a period N/L and contains one original and L-1 copies. Note that this function exhibits the same periodic behavior with copied spectrum, C(f,K). The division of C(f,K) with $D_z(f/L, K/L)$ eliminates (deconvolves) the periodicity (see Figure 1).

In summary, the Fourier transform of interpolated data, G(f,K), can be obtained by filtering (point-by-point multiply) the copied original spectrum, C(f,K), with an interpolation filter obtained from the data itself:

$$G(f, K) = H(f,K)\, C(f,K) = [\, D(f/L, K/L) / D_z(f/L, K/L)\,]\; C(f,K)$$

Note also that the interpolation filter acts like an on-off switch; where there is alias it tends to be zero, and where there is no aliasing it tends to be one (or L, depending on how it is exactly implemented). For 3-D data one has to add another variable corresponding to the second wavenumber[1,2] and padding or masking operations has to be done in all space directions. For example the copied spectrum will have one original and L^2-1 copies. The denominator portion of the operator will also contain one original and L^2-1 copies and acts like a deconvolution operator as in the 2-D case.

REFERENCES

[1] Gulunay, N. and Chambers, R. E., 1997, Generalized f-k domain trace interpolation, 67^{th} Ann. Internat. Mtg. Soc. Exp. Geophy., Expanded Abstracts 1100-1103.

[2] Gulunay, N. and Chambers, R. E., Unaliased spatial trace interpolation in the f-k domain, 1997, US Patent 5,677,892.

APPENDIX 1- Fourier transforms of a zero inserted sequence

The Fourier transform of length NL for a periodically zero inserted sequence of original length N (L-1 zeroes between each pair samples brings it to length NL) can be obtained from the Fourier transform of the original data by copying length N Fourier transform L-1 times.

APPENDIX 2- Fourier transform of a zero padded a sequence

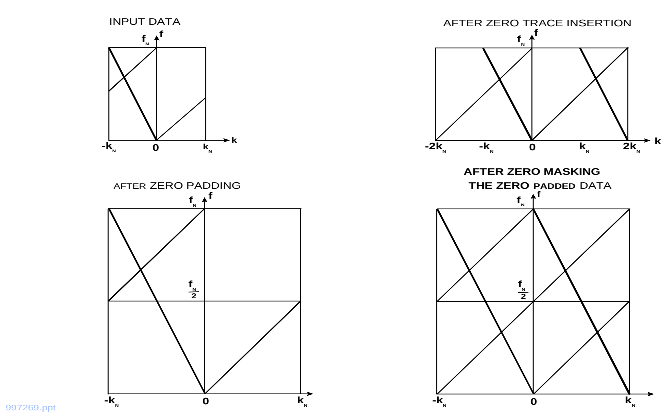
The Fourier transform of length NL of the zero padded sequence of original length N (the number of zeroes is (L-1)N) is an interpolated version of the original transform and every L-th sample of this spectrum has the same value with the original spectrum.

APPENDIX-3 Fourier transform of zero masked sequence

The Fourier transform of length N of the periodically zero masked (one live, L-1 zeroes, one live, L-1 zeroes, and so on) sequence of length N is a L times wrapped version of the original Fourier transform and is repetitious (it contains L periods each of length N/L). The Fourier transform of such a zero masked sequence can be obtained from the original Fourier transform of length N by adding L-1 copies of it to itself after index shifting each one by a multiple of N/L. Note that when an index exceeds its original range, it is reduce by N (the transform is wrapped).

FIGURE 1

D(f,k) C(f,K)



D(f/2,K/2) $D_z(f/2,K/2)$

FIGURE 2