Generalized f-k domain trace interpolation

N. Gulunay and R. Chambers; Western Geophysical*

SUMMARY

We have recently demonstrated that two-to-one trace interpolation for aliased events for 2-D data can be achieved by working in the f-k domain of the input data. Here we extend the method to interpolation factors higher than two as well as to 3-D data by working in the f-k domain of the output data.

INTRODUCTION

The need for trace interpolation prior to multichannel processes is well established. An efficient two-to-one trace interpolation for 2-D data was developed by Gulunay and Chambers (1996) by making use of the f-k domain and the sparse plane wave content of the input data. The f-k domain was chosen because of efficiency. This interpolator was a forward directional interpolator. The forward-backward interpolation operators presented by Gulunay and Chambers (1997) have less interpolation artifacts and can be merged with sinc trace interpolation. In this paper we extend the f-k domain trace interpolation to interpolation factors higher than two and to 3-D data by working in the f-k domain of the output instead of the f-k domain of the input.

THEORY

Suppose we wish to make a L to 1 trace interpolation where L is an integer;. that is we wish to interpolate L-l traces between each pair of traces. The number of traces after interpolation will be L times larger than the original number of traces and the trace distance will be L times smaller than the original trace distance. Note that wavenumber increment does not change during interpolation since the maximum offset of the data does not change. If N represents the number of traces in the input (after zero padding to the length required by spatial FIT), then there are N wavenumbers before the interpolation. After the interpolation, the wavenumbers in the spatial Fourier transform increase by a factor of L. We therefore expect the maximum wavenumber, k_N, of data to increase by a factor of L. That is, this process, if done correctly, will extend the spatial bandwidth of data by a factor of L. This can be done by first creating interpolated traces and then interleaving them with the original traces. If interpolated traces are correct, then interleaving them with the original traces unwraps the spectrum L-l times and provides the spectral extension.

For L=2, one can use the ratio of the f-k transforms of the even and odd numbered traces of the known record at half the temporal frequency (Gulunay and Chambers, 1996) to interpolate the traces. Going to odd numbered or even numbered traces doubles the trace distance and wraps the spectrum once. Since the wavenumber increment stays the same, the Nyquist wavenumber of the odd or even numbered traces is halved. The operator, H, that produces the interpolated (unknown) traces from the known data, D, by multiplication in the f-k domain

$$U(f, k)=H(f, k)D(f, k)$$

is obtained by taking the even and odd numbered traces of the known data (with no zero traces) as a model and then by making use of half the temporal frequency. This is done because we assume a limited number of plane waves (dipping events) exist in the data. For a plane wave at frequency f, interpolated and original traces relate to each other the same way that the even and odd numbered traces of the original record do at half the temporal frequency, f/2. More specifically, the interpolation operator can be obtained by spectral division

$$H(f, k)=B(f, k)/A(f, k)$$

where numerator and denominator are stretched Fourier transforms of the even and odd numbered traces of the known gather at half the temporal frequency:

$$B(f, k)=D_{even} (f/2, k/2) \text{ and } A(f, k)=D_{odd} (f/2, k/2).$$

Since f-k spectra of the odd and even numbered traces are expected to be the same, magnitude of the operator, H, can safely be set to one leading to an all-pass operation in the f-k domain of the input. When a zero trace is inserted between each pair of traces, we see a replication of the f-k spectrum along the k direction. Comparing this with the f-k spectrum of the data interleaved with the interpolated traces obtained from the inverse transform of U(f, k) given above, we observe that the net result in the f-k domain of the output is certainly not allpass, but a data adaptive alias suppressor. Note that placing a zero trace at every trace location where we need an interpolated trace produces an f-k spectrum which is periodic along the k direction with a period of N (corresponds to $2k_N$). What is then the interpolation operator to convert the zero trace inserted record to fully interpolated record? In other words, we wish to find an operator, 0, such that

$$G(f, K) = O(f, K) C(f, K)$$

where G(f, K) is the f-k transform of the full (interpolated) gather, K=Lk, is the new wavenumber, C(f, K) is the f-k transform of the zero trace inserted gather. Note that C is a

$$C(f, k+m2k_N)=D(f, k) \quad m=0,1,2,..,L-1$$

For L=2, the output domain based operator, 0, can be related to the input based operator, H. For a single event with a slope of At seconds per trace, H can be shown to be

$$H(f, k)=\exp(j\,2\pi f\,\Delta t/2).$$

For these data, C(f, K) contains two linear events, one originating from the origin (O,O), the other from (0, $2k_N$.) The operator can be shown to be

$$O(f, K)=1+\exp(j\,2\pi\,(K/4k_N+f\,\Delta t/2)).$$

Note that this operator takes the value of 2 at the peak of the dipping event and becomes zero when one moves away from it by $2k_N$ along the k-axis. That is, it becomes zero at the alias locations on C(f, K), hence the alias suppression.

When L>2 we cannot form odd and even traces and it may not be easy to find operator H. Then how do we derive the operator 0 directly? This we do by taking the original record and its masked version at L times lower temporal frequency as model. In other words, we do the spectral division:

$$O(f, K)=A(f, K)/B(f, K)$$

where

$$A(f, K)=D(f/L, K/L)$$

is the stretched (interpolated) version of the original f-k spectrum, D(f, k), at L times lower frequency and

$$B(f, K)=D_z(f/L, K/L)$$

is the stretched version of the Fourier transform of the zero masked original data (hence the subscript, z) at L times lower frequency. For example, when L=3, D_z is the Fourier transform of the original record after zeroing traces 2,3, 5,6, 8,9... which corresponds to multiplying the input traces with a combing function

$$w(x)=(1,0,0,1,0,0,....).$$

Since multiplication in x is a convolution process along the K axis and W(K), the Fourier transform of w(x), is also a combing function, B(f, K) can directly be calculated from A(f, K) by laterally shifting by an integral multiple of $2k_N$ and summing it onto itself, L times. During the calculation of B from A modulo $2Lk_N$ is understood, that is, as the wavenumber in A exceeds its range, $(-Lk_N, Lk_N)$, it is wrapped back.

After inverse Fourier transforming G(f, K), a gather is obtained with traces at original as well as zero trace locations. We replace interpolated traces that are at the original trace locations with the original traces. This way we preserve the original traces in the final output and also provide visual quality control since interpolated traces and original traces are side by side and the interpolation errors are now easy to detect.

Extension of this method to 3-D was implemented as follows. Assume that the input has N_x traces along x-axis and N_y traces along y-axis. We desire LN, traces along x-axis and LN_y traces along y-axis after interpolation. Let D(f, k_x ,k_y) represent the f-k transform of the input. Then the f-k transform of the output data is

$$G(f, K_x, K_y)=O(f, K_x, K_y)C(f, K_x K_y)$$

where C is the L times copied version of D along each axis, k_x and k_y and

$$O(f, K_x, K_y)=A(f, K_x, K_y) / B(f, K_x, K_y)$$

where

$$A(f, K_x, K_y)=D(f/L, K_x/L, K_y/L)$$

is the stretched original spectrum and

$$B(f, K_x, K_y)=D_z(f/L, K_x/L, K_y/L)$$

is the stretched version of the Fourier transform of the zero masked data. Zero masking is done along the x and y-axes with the same mask. Note again that 1/L in these expressions necessitates interpolation (stretch) along temporal frequency as well as each wavenumber direction. As before, B can be obtained by summing A onto itself after shifting it laterally L-1 times along each axis, each time with a shift of $2k_{Nx}$ along K_X and $2k_{Ny}$ along K_y directions where k_{Nx} and k_{Ny} are the Nyquist wavenumbers of the original data. 3-D f-k interpolation results will be shown during the presentation..

PRACTICAL CONSIDERATIONS

The following issues should be considered when interpolating in the f-k domain:

- The interpolator, described above, can be applied prestack or poststack. It assumes that amplitudes of the events are balanced. To reconstruct the missing shots during data acquisition prestack gathers have to be either CMP gathers or common receiver gathers. The curvature of the data must be reduced and dips are centered around zero by applying a nominal NMO.

- At lower frequencies or over areas with small dips there is no need to calculate the operator with the procedure described above. Given a maximum dip, a frequency value can be calculated below which a sinc filter with a boxcar response is adequate. At such frequencies we do sinc interpolation. Sinc interpolation reduces the cost and is more accurate than relying on lower frequencies when the frequency of interest is not wrapped around.

- Variation in moveout with offset can be exploited when operating on CMP data. This means that inner offsets of a CMP gather can be interpolated with sinc at almost all frequencies.

- The expected maximum amplitude of the operator, O(f, K), is L. Values higher than this are most likely due to a small denominator and can be clipped to L.

- The operator O(f, K) has both an amplitude and phase spectrum. As cast above it acts like a forward interpolator. By selecting which traces to zero in the input data in obtaining the denominator, the operator can be made symmetrical in the t-x domain. For L=3, w(x)=(0,1,0,1,0,1,0...) produces almost a symmetrical (o(-t,-x)=o(t, x)) operator in the t-x domain. If Fourier transforms of zero masked traces, B, are obtained from A as before, this can be done by phase shifting each component proportionally to the lateral shift before summation. Even then, noise might cause the operator not to be exactly symmetrical in the t-x domain. Although it is possible to force it to be symmetrical by manipulating its f-k spectrum, we find that bringing the operator to t-x domain, applying a symmetrical taper, and taking it back to f-k domain produces more reliable results.

- Zero traces might need to be padded to the data to satisfy FFT input size requirements and to eliminate lateral wraparound. Since this process relies heavily on the linearity and sparseness of the events in the t-x domain, spatial and temporal gating is generally required. A vertically and laterally sliding window scheme with a capability of discarding the few traces at the edges of the space window may need to be used.

- As in any frequency domain deconvolution process small values in the denominator need to be detected and precautions taken. We find one percent of the peak amplitude of the f-k spectrum to be an adequate threshold for this.

- For steeply dipping data like ground roll one might be tempted to interpolate many new traces between each pair of input traces to improve the preprocessing. Large interpolation factors not only increase the volume of data to be processed but are less accurate. A large L means that frequencies lowered by a factor L will be used for operator design. Since the low end of the seismic spectrum is generally missing or falls into the taper zone of the field filter, we find large L values not producing reliable operators at low frequencies. We find L=2 and 3 to be most likely applications of this technique. L=4 can be achieved by a cascaded run of two L=2 runs and seems to be more reliable than a straight L=4 run. A possible explanation for this is that two L=2 runs rely less heavily on the low frequency information than a single L=4 run.

EXAMPLES

Figure 1 and 2 illustrate a CMP gather before, and after 3: 1 trace interpolation. The dataset is from the Gulf of Mexico and has an offset extending to 8000 m offset. Therefore water bottom multiples, have a large moveout. Data was NMO corrected with a velocity between multiple and primary trends so that both primaries and multiples are in reasonable dip range. Dips at the far offsets are about 50 ms per trace before interpolation. The technique described above is used to do the interpolation except at low frequencies and low dips (inner offsets) where sinc interpolation is adequate. A set of 12-trace 400 ms windows overlapping 100 ms in time was used for the interpolation We observe that 3: 1 interpolation of the primaries and multiples is adequate.

CONCLUSIONS

We have illustrated a general method of interpolating aliased seismic data using only the f-k domain. The method is general in the sense that it allows interpolation factors greater than two and is also applicable to 3-D data.

ACKNOWLEGMENTS

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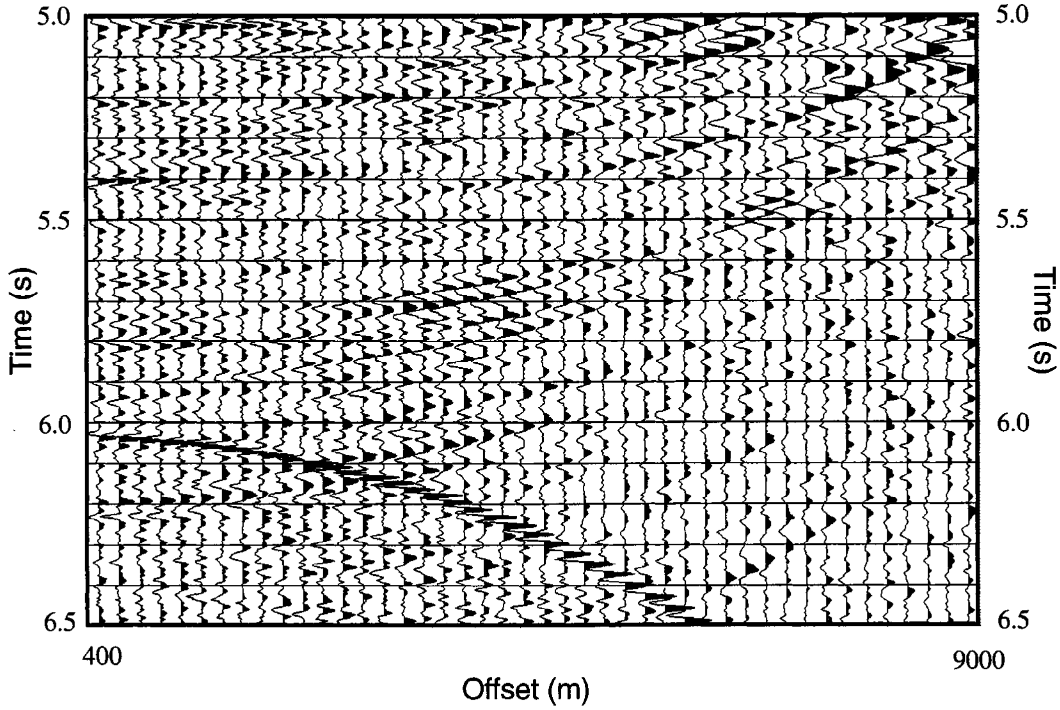

Figure 1. A CMP gather from the Gulf of Mexico. NMO with a velocity function between primary and multiple trends was applied.

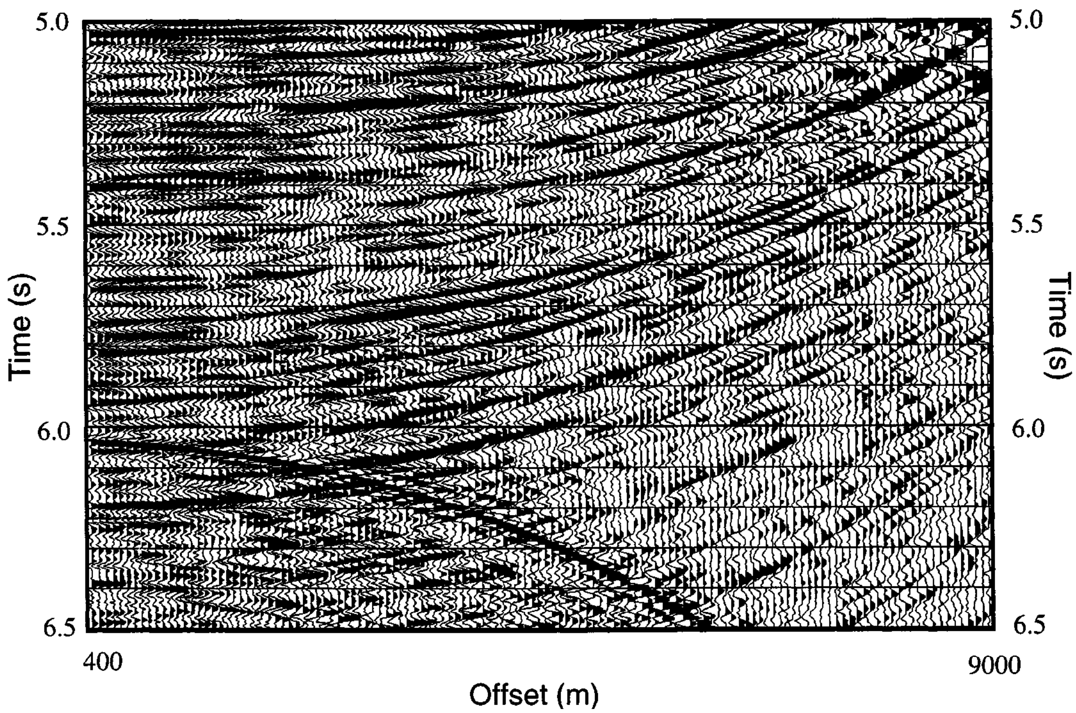

Figure 2. The CMP in Figure 1 after 3:1 f-k domain trace interpolation.