Seismic Processing: Radon Transform and Multiple Elimination

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F-X Domain Least-Squares Tau-*P* and Tau-*Q* — SP1.1

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SUMMARY

The Tau-Q transform which is based on residual moveout and stack is a parallel process to the Tau-P transform which is based on linear moveout and stack. Both of these processes can be implemented in the f-x domain. In this domain, residual moveout involves complex exponentials with arguments which are quadratic in offset while linear moveout involves arguments which are linear in offset. When the coefficients of the transforms at each frequency are obtained through least squares error constraint rather than through straight sums in the frequency domain, a best fit is obtained to the dips or the parabolic residual moveouts that are assumed to exist in data. I will use the terms "F-x Domain Least Squares Tau-Q" or "Radon Tau-Q" for one and "F-x Domain Least Squares Tau-P" or "Radon Tau-P" for the other.

The resolutions of the Radon Tau-P and the Radon Tau-Q are better than those of the classical Tau-P and the classical Tau-Q only when the problem at hand allows the amount of white noise used in the Wiener-Levinson Inversion to be small. The side lobes of the classical residual normal moveout and stack are related to the Fresnel Integrals and are significant in magnitude. The least squares error nature of the Radon Tau-Q suppresses these side lobes significantly. Therefore, the Radon Tau-Q gathers are cleaner than those of the classical Tau-Q.

INTRODUCTION

The classical Tau-P method is well established (Phinney et al.(1981), Tatham(1984)). In simple terms, it is linear moveout followed by stack (slant stack). A convolutional operator known as the "Rho filter", which is equivalent to a linear ramp in the frequency domain balances the spectrum of the forward transform. A common application of Tau-P transform is to decompose seismic data into various dip components.

Thornson(1984) imposed the least squares error constraint on the reconstructed data and developed "Slant Stack Stochastic Inversion". He also generalized stacking velocity decomposition into "Velocity Stack Stochastic Inversion". Hampson(1986) was able to implement Thornson's method efficiently by using NMO corrected data, the parabolic approximation to residual moveout and the f-x domain. Hampson's technique imposes the least squares error constraint on the model constructed for the data once for each frequency. Hampson(1987) identified his approach as the Discrete Radon Transform which is explored by Beyklin (1987). In what I call the "Radon Tau-P", I use parameter "p" for linear moveout and refer to Hampson's method as the "Radon Tau-Q" where q is the parameter for residual (parabolic) moveout:

$$t = tau + p * x \quad \text{(linear move-out)} \qquad \text{(Eq. 1)}$$

$$t = tau + q * x * x \quad \text{(residual move-out)} \qquad \text{(Eq. 2)}$$

In the Radon Tau-P method I find slant stack inverse in the f-x domain as in Hampson's method. The base functions in this approach become $e^{j\,w.p.x}$ in stead of $e^{j\,w.q.x.x}$. Because of its least square nature I will often refer to this technique as the "F-x domain least squares Tau-P".

THE SLANT STACK IN THE F-X DOMAIN

Time domain implementations of the Tau-p transform require interpolating data from discrete time samples to the time values implied by Eq. 1. In the forward transform, we are in effect applying linear moveout by the amount -p.x and summing the results and dividing by number of elements in the sum. Since a static shift is a linear phase addition to the data in the frequency domain, the frequency response of Tau-P trace at angular frequency w will be

$$g(w,p) = \frac{1}{Nx} \sum_{k=1}^{Nx} D(w,x_k) \,.\, e^{-j\,w.p.x_k} \qquad \text{(Eq. 3)}$$

where Nx is the number traces in the data, x_k are the offsets, and $D(w,x_k)$ is the Fourier transform of kth trace at angular frequency w. The inverse Fourier transform of g(w,p) is the slant stack trace or Tau-P trace.

The resolution of the Tau-P process can be studied as a function of frequency if we use a flat event at time t0 :

$$g(w,p) = e^{j\,w.to} \,.\, a(w,p) \qquad \text{(Eq. 4)}$$

where

$$a(w,p) = \frac{1}{Nx} \sum_{k=1}^{Nx} e^{-j\,w.p.x_k} \qquad \text{(Eq. 5)}$$

If the trace distance increment is constant and is dx, then one can show that the magnitude of this function is given by

$$|g(w,p)| = \frac{|\sin(Nx.w.s/2)|}{Nx.|\sin(w.s/2)|} \quad \text{(Eq. 6)}$$

where $s=p.dx$ is the slope (dip) per trace.

The function given in Eq. 6 occurs in many branches of science. For example, the magnitude of diffracted light from a grating of Nx elements with element width dx is given by the same formula where s is a parameter related to the angle measured from the direction of the incident light beam. If viewed as a function s, this function determines the resolution of the process. To find the half power point of the response involves trigonometric equations. Instead, the first zero crossing value given by

$$Nx.\,s_0 = \frac{1}{\text{frequency}} \quad \text{(Eq. 7)}$$

can be used. Note that Nx.s is the moveout at far offset.
The plot of Eq. 6 as a function of Nx.s at 15 Hz is given in Figure 1. Nx.s0=66 ms is where the first zero crossing occurs. Two events with "s" value difference (say dip difference) less than s_0 are considered to be unresolvable. This is known as "The Rayleigh's Criterion" in Optics (Born and Wolf ,1980).

THE F-X DOMAIN LEAST SQUARES TAU-P (THE RADON TAU-P)

When Hampson's (1986) approach is used for Tau-P, a system of normal equations is obtained at each frequency for N unknown coefficients $f(p_i)$ i=1,,...,N,

$$\sum_{j=1}^{N} R(w,p_i,p_j).f(p_j) = g(w,p_i) \quad \text{(Eq. 8a)}$$

where N is the number of dips the data are assumed to contain.
I make the observation that the right hand side of Eq. 8a is the classical Tau-p response at angular frequency w. The R matrix on the left hand is independent of the data, and serves as a denominator in a sense:

$$\underline{f} = R^{-1} . \underline{g} \quad \text{(Eq. 8b)}$$

The role of R^{-1} is to sharpen the classical Tau-P response.

When the dip increment at a particular angular frequency is chosen to be constant, then the matrix becomes Hermitian Toeplitz with main diagonal all ones and nth lower diagonal given by

$$R_n = \frac{1}{Nx} \sum_{k=1}^{Nx} e^{-j\, n.w.dp.xk} \quad \text{(Eq. 9)}$$

and the linear system of equations given in Eq. 8a can be efficiently solved (Kostov, 1989).
Important observations are:
a). Rn diminishes in magnitude as offset range=Nx*dx goes to infinity. Then, the R matrix reduces to a unit matrix. That is, the infinite aperture limit of the Radon Tau-P is equivalent to the classical Tau-P,
b). Rn array is the same with the $\underline{g}$ array of the flat event given by Eq. 5.
Therefore Eq.8b has to return a perfect solution (a spike) for the $\underline{f}$ array. That is, the Radon Tau-P can produce the ideal solution, "infinite aperture solution", from finite apertures if the matrix to be inverted is not singular.

EFFECTS OF WHITE NOISE ON THE RADON TAU-P

Various conditions cause the R matrix to be singular (Kostov, 1989). An obvious one is at zero Hz. In this case the matrix becomes all ones and is impossible to invert. To decrease the arithmetic problems caused by such singularities, it is common to add some white noise to the main diagonal of the R matrix, changing it from $R_0=1$ to $R_0=1+n$ where n is a small positive value.
To eliminate the amplitude loss this modification will cause on the solution array $\underline{f}$, I multiply the right hand side of Eq. 8a with 1+n. With this scheme I obtain an algorithm which gives $\underline{f} = \underline{g}$ as n goes to infinity. Therefore the Radon Tau-P reduces to the classical Tau-P as the added white noise goes to infinity.

When the matrix is not singular and the white noise can be chosen to be small, then this is the best solution (in the least square sense) that can be constructed from the assumed dips.

To understand the effect of white noise on the Radon Tau-P(or Tau-Q) solution I use the same record that contains a flat event and model it in terms of two dips: one is the correct dip ($dip_1=0$) the other one is an incorrect one ($dip_2=s$). The normal equations become

$$\begin{vmatrix} 1+n & a^* \\ a & 1+n \end{vmatrix} . \begin{vmatrix} f_1 \\ f_2 \end{vmatrix} = e^{j\, w.to}.(1+n) \begin{vmatrix} 1 \\ a \end{vmatrix}$$

where a is given by Eq. 5 and a^* is its complex conjugate.

The determinant of the matrix is

$$D=(1+n).(1+n) - a^*.a$$

and the analytical solution for the Radon Tau-P can be shown to be

$$f_1 = e^{j\, w.to}.(1+n-a^*.a).(1+n)/D$$
$$f_2 = e^{j\, w.to}.(1+n).n.a / D$$

I make three observations from this simple case:
a). As the white noise parameter goes to infinity, I obtain the expected classical Tau-P solution

$$(f_1,f_2) = e^{j\, w.to} .(1,a).$$

b). As the white noise parameter goes to zero, I obtain the ideal solution

$$(f_1,f_2) = e^{j\,w.to}.(1,0).$$

For N dips we would get

$$(f_1,f_2,..,f_N) = e^{j\,w.to}(1,0,0,...,0).$$

That is, no energy leaks from the correct dip (dip=0) to the incorrect ones when a zero value can be used for the white noise parameter.

Figure 2 illustrates this point using a moderate (1%) and a small (0.01%) white noise value. Note the resolution increase in both cases. However, when the matrix is singular which happens often in practice, any moderate noise used makes the Radon Tau-P results almost identical to the Classical Tau-P.

c). At zero Hz, $a^*.a = 1$, and I obtain

$$f_1 = f_2 = e^{j\,w.to}.(\,(1+n)/(2+n)\,).$$

It can be shown that at zero Hz, for N dips we get

$$f_1=f_2=...=f_N= e^{j\,w.to}.(\,(1+n)/(N+n)).$$

This means that it is not possible to determine to what dip DC energy belongs and therefore the Radon Tau-P distributes that energy equally between all dips.

It can also be shown that as the number of dips used to model the input data goes to infinity, the spectrum of the zero dip trace becomes a line that passes through 0 at zero Hz and 1 at Nyquist frequency implying that infinite dip limit of Radon Tau-P is the Rho filtered Tau-P.

THE RESIDUAL MOVEOUT AND STACK IN THE F-X DOMAIN

The residual moveout after NMO, followed by stack can be done through the f-x domain parallel to Eq.3:

$$g(w,q)=\frac{1}{Nx}\sum_{k=1}^{Nx} D(w,x_k).\,e^{-j\,w.q.x_k x_k} \qquad \text{(Eq. 10)}$$

where $D(w,x_k)$ represent the Fourier transform of the NMO applied data at offset x_k. The inverse Fourier transform of $g(w,q)$ is the RNMO+stack trace to which I will refer as the Tau-Q trace.

The resolution of the Tau-Q trace can be studied as a function of frequency or moveout if we apply it to the same flat event used above (q=0 event). Then

$$g(w,q)=e^{j\,w.to}\,.\,b(w,q) \qquad (\text{Eq.11})$$

$$b(w,q) = \frac{1}{Nx}\sum_{k=1}^{Nx} e^{-j\,w.q.x_k x_k} \qquad (\text{Eq.12})$$

This sum behaves like the Fresnel Integrals

$$\int_0^V \cos(\pi.U^2/2)\,dU \quad \text{and} \quad \int_0^V \sin(\pi.U^2/2)\,dU$$

which are used in Optics for the diffraction of light from a straight edge(Born & Wolf, 1980). In our problem, we get

$$V = (NH-1).dU$$
$$dU = (\,2.f.q.dx.dx)^{1/2}$$

and NH=Nx for offend shooting and NH=Nx/2 for split spread. Since Fresnel integrals oscillate around the limit value of 0.5 as V goes to infinity I find that the sum in Eq. 12 has a frequency and moveout dependent limit:

$$\frac{1}{Nx.dx.\,(\,f.\,q\,)^{1/2}} = \frac{1}{(\,f * dT\,)^{1/2}}$$

where dT is the residual moveout at far offset: $dT = q\,.\,(Nx.dx).(Nx.dx)$.

The plot of Eq. 12 at 15 Hz and as a function of dT is shown in Figure 3. Note the significant side lobes in the figure. Only at high frequencies or at significantly different moveouts than where the event is, or for large offset ranges (large spatial apertures), the bias in the oscillation point will be small. Otherwise, the classical Tau-Q will have significant side lobes explaining why a single event shows up at many velocity panels in standard CVS panels.

THE F-X DOMAIN LEAST SQUARES TAU-Q (THE RADON TAU-Q)

When the least squares error constraint is imposed on the problem, we obtain Radon Tau-q. When the q increment is kept constant, we obtain a set of normal equations with Hermitian Toeplitz form as before with the only difference that parameter q takes the place of the parameter p and x.x takes the place of x. Similar arguments lead to the conclusions

a. The large aperture limit of the Radon Tau-Q is the Classical Tau-Q.

b. The high white noise limit of the Radon Tau-Q is the Classical Tau-Q.

c. The energy at zero Hz is equally shared between all curvatures and therefore that value goes to zero as the number of parabolae used in the model goes to infinity.

Figure 4 compares the Radon Tau-Q to the Classical Tau-Q using a moderate (1%) and a small (0.01%) white noise value. Note the resolution increase and the side lobe suppression in both cases. When the matrix is singular which happens often in practice, a moderate noise value needs to be used. I observe that even in singularity case the side lobe suppression takes place in Radon Tau-Q, even though the resolution is no different than the classical Tau-Q.

CONCLUSIONS

The least squares Tau-P and the least squares Tau-Q can be implemented in the f-x domain parallel to each other. The infinite aperture or infinite white noise limit of both processes are their classical counterparts,i.e., the slant stack, and the residual moveout & stack. When the matrices to be inverted are non-singular, both methods eliminate the smearing effect that finite apertures cause, giving highly resolved (infinite aperture type) results. When matrices are singular, which happens often in practice, the Radon Tau-P gives results almost identical to the classical Tau-P. On the other hand, the Radon Tau-Q with moderate noise suppresses side lobes even in singularity cases, yet with no improvement in the resolution.

ACKNOWLEDGMENTS

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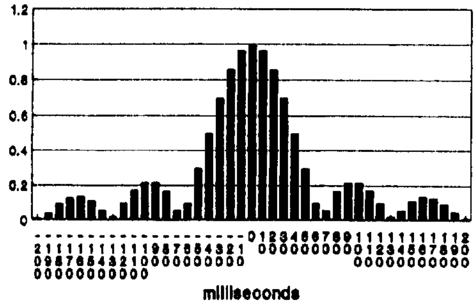
FIGURE 1. Classical TAU-P (Magnitude)
as a func.of lin.moveout at far offset



dx:500 ft, Nx:13, freq:15 Hz, Xm:6000 ft

FIGURE 2. RADON TAU-P Response
Magni. vs. Lin. moveout at far offset

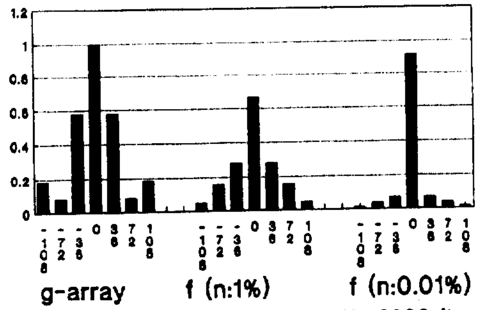


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FIGURE 3. Classical TAU-Q (Magnitude)
as a func. of res. moveout at far offset

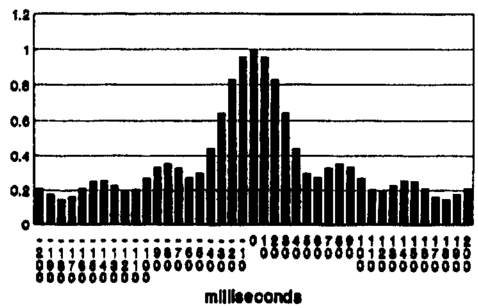

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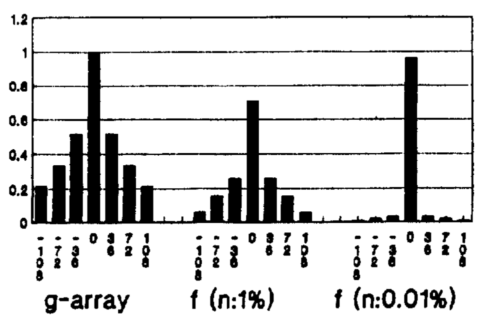
FIGURE 4. RADON TAU-Q Response
Magni. vs. Res. moveout at far offset



dx:500 ft, Nx:13, freq:15 Hz, Xm:6000 ft